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Reliability Trend Analyses With Statistical Confidence Limits Using the Luke Reliability Trend Chart

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WITH STATISTICAL CONFIDENCE LIMITS USING
THE LUKE RELIABILITY TREND CHART**

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ABSTRACT

RELIABILITY TREND ANALYSES WITH STATISTICAL CONFIDENCE LIMITS USING THE LUKE RELIABILITY TREND CHART

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In electronic systems, it is interesting to understand exactly how the reliability is changing with time. Dynamic performance changes when a system passes from infant mortality stage into useful life phase and when the system passes from useful life phase into wearout phase. Dynamic performance also changes when the system is redesigned or when the system is acted on by a number of other outside forces such as a change in maintenance policy, escalation of alignment problems, or a change in training program. It is important to know when a system is changing dynamically in order to assess design, policy and program changes and to determine when changes in life cycle phase are occurring.

This study presents a methodology to analyze the reliability of electronic systems as they change in time dynamically. The method is developed mathematically and is proven with a simulation to be able to estimate system MTBF and to be able to determine when process changes occur. Three case studies of problem power supplies are provided to illustrate how the technique has been used to make cost avoidance decisions.

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CHAPTER 1

INTRODUCTION

Background

In Reliability Engineering, electronic equipment performance is usually statistically measured in terms of failure rate or Mean Time Between Failure (MTBF). Although the two values yield identical information (one is the reciprocal of the other), they are used for different purposes. Failure rate is an indication of how many failures are likely to occur per some unit of time, usually expressed in failures per million hours. MTBF, which is expressed in hours, is an indication of how long a system can operate before a failure is likely to occur. Usually, analysts prefer to use MTBF when discussing a system's performance and prefer to use failure rate when referring to part performance. Analysts use failure rate for parts because individual parts may fail so infrequently that their MTBF values are extremely high and are difficult to conceptualize. Analysts use MTBF for systems because MTBF is easier to conceptualize than failure rate when MTBF is small. The failure rate and the MTBF are usually determined statically (that is once for all available data). Therefore, they can only be analyzed after a significant amount of data are available. These measures are interesting statistically because electronic equipment is modelled as exponentially distributed

(MIL- HDBK-338-IA 1988) with a mean equal to MTBF and a standard deviation equal to MTBF^2.

In most systems, it is important to understand exactly how the system is performing dynamically. Dynamic performance changes when a system passes from infant mortality stage into useful life phase and when the system passes from useful life phase into wearout phase. Dynamic performance also changes when the system is redesigned or when the system is acted on by a number of other outside forces such as a change in maintenance policy, escalation of alignment problems, or a change in training program. It is important to know when a system is changing dynamically in order to assess design, policy and program changes and to determine when changes in life cycle phase are occurring.

There are some notable methods to assess reliability of parts and systems in the literature. Basically, these methods fall into the following groups: Sequential Life testing, Reliability Growth testing, Posterior testing, and Statistical Process Control (SPC). Sequential Life testing analyzes whether, with given confidences, a system should be rejected, accepted or tested until a suitable conclusion can be reached. Sequential Life testing is neither designed to provide an accurate estimate of reliability nor designed to detect when changes are introduced into a system. Reliability Growth models show whether system reliability is increasing or decreasing with time. As such, they are able to detect gradual shifts in the reliabilities of systems. Reliability Growth models do not adequately predict step changes to reliability which are characteristic of changes in the field. Posterior tests use existing (*a priori*) values of reliability parameters, combine them with fleet values, and then form a posterior value. The strength of the method is that it utilizes the maximum amount of data possible. The same strength becomes its weakness in the presence of a process change because the new field values are diluted by the *a priori* values. SPC can be used to analyze reliability level and identify changes in reliability. Unfortunately, because of the shape of the exponential distribution, SPC techniques require a great number of test points to have any confidence in the results. SPC often cannot be used because there are too few data points for analysis.

Currently, there is not a method which can demonstrate changes in reliability of a system or part within a suitable small set of data. There needs to be a method which both measures precisely the actual reliability of a system or part and also detects when a perturbation to that system due to change in design, use or logistics causes a change to reliability. The method must be able to give results with statistical confidences so that analysts can decide on the degree of confidence of the results.

In this research, an analysis method is introduced called the Luke Reliability Trend Chart (LRTC). The LRTC analyzes reliability trends and parameters of exponentially distributed electronic equipments as they change over time. The basis for the LRTC is similar to SPC except that the LRTC does not rely on the Central Limit Theorem since the shape of a reliability distribution for electronic equipment is already assumed to be exponentially distributed.

This research applies only to electronic reliabilities which can be modeled as exponential distributions. However, there is potential to expand the results to the wider family of Weibull Distributions as well as Normal and other distributions.

Research Objective and Approach

In reliability engineering, traditional approaches to analysis of electronic equipment which is under change is inadequate. The objective of this research is to develop, demonstrate, and evaluate an approach to analyze electronic equipment under change. The research approach would address the following issues:

1) Estimate reliability parameter MTBF.
2) Detect shifts in reliability over time.
3) Monitor reliability of system for stability.

To accomplish the above objectives, this research first establishes the mathematical model for the LRTC. Secondly, a simulated exponential distribution which has a stable MTBF is generated and tested for whether the LRTC can estimate the mean without giving a false indication of a shift in MTBF. Next, a distribution is generated with a shift to determine whether the LRTC can detect the shift. Finally, the LRTC is compared to other techniques, namely, the Sequential Life test, The Duane Reliability Growth model, Posterior test and SPC techniques. The following hypotheses are formulated to test the LRTC:

$H1_1$: Does the LRTC technique describe the system parameter , MTBF, of systems with exponentially distributed hazard functions with minimum error?

$H0_2$: Is the LRTC equally likely to report a change in a process when no change has occurred (Type I error) as the Sequential Life test?

$H0_3$: Is the LRTC equally likely to make a Type I error as the Duane Reliability Growth model?

$H0_4$: Is the LRTC equally likely to make a Type I error as the Posterior test?

$H0_5$: Is the LRTC equally likely to make a Type I error as the SPC Techniques?

$H0_6$: Is the LRTC equally likely to report that no change has occurred in a process when a change has actually occurred (Type II error) as the Sequential Life test?

$H0_7$: Is the LRTC equally likely to make a Type II error as the Duane Reliability Growth model?

$H0_8$: Is the LRTC equally likely to make a Type II error as the Posterior test?

$H0_9$: Is the LRTC equally likely to make a Type II error as the SPC Techniques?

CHAPTER 2

LITERATURE REVIEW

A comprehensive literature review was conducted to assess other methods used to analyze reliability of electronic systems under change. This chapter is divided into six parts. The first part, point estimates of reliability, establishes that there is a need expressed in the literature to estimate MTBF within statistical boundaries. The literature reviewed treats reliability statically and could be improved if dynamic relationships were also considered. The second section, Reliability Growth models, studies the available literature on techniques used to demonstrate reliability growth, usually in production. These methods treat reliability as a dynamic function and seek to explain its characteristics in terms of its changing nature. The third section investigates methods to estimate field stress time when all stress time is not available. The forth section researches the work which estimates reliability parameters using a Bayesian synthesis of *a priori* and field distribution to produce a posterior distribution. The fifth section establishes the application of traditional SPC techniques in estimating reliability parameters and monitoring the change in reliability parameters over time. The sixth and final section draws conclusions from all five sections and shows the need for the LRTC.

Point Estimates of Demonstrated Reliability

Computer systems have evolved from low inherent availabilities of 60% or less in the 1950s to inherent availabilities of 90% or more in the 1980s and inherent availabilities over 99% in the early 1990s. This is still inadequate for certain industries which require 99.999% or higher availability. 99.999% inherent availability translates to five minutes of downtime per year. Perera (1993) illustrates how to theoretically show an increase of reliability and availability using a redundancy technique with mirrored disks. Without mirroring, test systems achieved 1.57 kHrs MTBF and 99.329% availability. With Mirroring, the system was predicted to achieve 32.8 Khrs MTBF and 99.9892% availability. Using field results from four systems, the system actually produced a point estimate of 34.56 Khrs MTBF and 99.9855% availability. Unfortunately, the field results were inconclusive because there was insufficient test time to report with statistical boundaries. Furthermore, there was not an indication of the time sequence of the failures which indicate useful information regarding possible trends and causality.

Yang (1993) evaluates the performance of the Multi-chip Unit (MCU) and the High Density Signal Carrier (HDSC) which have been redesigned using new technology. The initial predicted MTBFs were 70A hours for the MCU and 1000A hours for the HDSC. The author used field data to substantiate results. According to the author, there was an average of 8000 MCU/HDSC units over a 24 month period which produced six confirmed failures. The author then computed a point estimate of MTBF of both chips to be 1346A hours. The author does not provide a confidence level, an exact time sequence of failure data, or a composition of the two different microchips. The author neither shows whether it is statistically verifiable nor provides a statistical analysis of the dynamic relationship of field changes to respective failures.

Zaino et al. (1992) describes an analysis of a run-in procedure used to assure that early latent failures can be removed from products before selling products. Run-in is more similar to Environmental Stress Screening than it is to Reliability Growth testing because it is used to accelerate failure rate outside of the infant mortality phase of life cycle. A run-in policy is the policy which dictates how many actuations or cycles a component or system is to be ran before selling. The authors compared the failure times for two control groups of systems which had been sent out to customers after being tested at the factory for 500 run-ins and for 5000 run-ins. The results were then compared using several methods to estimate rate of occurrence of failures. The results showed that increased run-ins caused a corresponding decrease in failure rate but at additional cost. The problem which the authors were trying to solve was to minimize the cost while maximizing reliability. The authors were attempting to estimate reliability as a function of run-in time. Therefore, they constructed confidence limits at $\pm 2\sigma$ from the point estimate of the reliability at a given point in time. Authors presented a nonparametric method to discover the differences between machines which was basically a χ^2 approach. The results were unable to fully account for the difference between the two run-in plans, but they did demonstrate some machines were operating uncharacteristically ("special causes"). This nonparametric method chosen was unable to give specific inferences about true values. Furthermore, it is not a dynamic method for analyzing data. A dynamic model would provide a statistical median estimate of each process with associated confidence of each estimate and would clearly show statistically which values were aberrant. Such a method would allow the analyst to study aberrant values more carefully.

Murray (1992) reports reliability data collected on an irregularly used system (the Strategic Petroleum Reserve). The author used the collected reliability data to better estimate the reliability of components in the future and to make recommendations for system improvements. The reliability estimates given for part performance in the paper are mean values. The paper reported that 1.3% of the failures were design related and that 45.9% of all failures were "wearout" related. However, the research excludes the analyses of either improvement due to redesign or beginning of wear-out phase.

Reliability Growth Models

Crow (1993) develops a method to estimate confidence intervals for the failure intensity function for repairable systems using a power law nonhomogeneous Poisson process model. He shows how to use this data to analyze burn-in (infant mortality), useful life and wearout. Crow is concerned with the life cycle parameters of generic identical systems, he superimposes failure data of all systems onto one timeline. This provides better data to analyze infant mortality, useful life and burnout. The superposition technique cannot be applied to assess programmatic or design changes in systems because different equipments are at different stages of lifecycle when programmatic changes or design changes are introduced. The failure intensity function is modelled as a nonhomogeneous Poisson process which is mathematically similar to the Weibull distribution. The difference is that a process is a function of system age and sequence of events whereas a distribution is a static representation of a single failure probability. MIL-HDBK-189 outlines a procedure to determine if the reliability of a system or component is statistically changing over time using the χ^2 distribution. MIL-HDBK-189 shows how to use that information to make inferences about whether a process is changed or not over time. It does not show dynamically how a system is in the process of changing over time.

Campodónico (1993) presents a computer program to assess reliability growth based on failure time data for a Weibull distribution. The model assumes that the natural logarithm of failure times is normally distributed and uses results in an auto regressive test for reliability growth or decay with respect to time. Similar to Crow (1993) and MIL-HDBK-189, Campodónico's method does not detect trends over time, but rather investigates whether or not system has already changed with respect to time. Method of integrating current failure distribution with prior assumed distribution is more rigorous and more difficult than Giuntini et al. (1993).

Demko (1993) shows that AMSAA and Duane models for reliability growth are unable to predict nonlinear reliability growth accurately. The author suggests a piecewise regression method be used. From the data presented in the paper, the piecewise method is actually oversensitive, showing downward trends when actual cause is probably data variation. Demko points out that for AMSAA and Duane models, undue weight is given to early results masking downward trends. The method espoused by the paper, piecewise regression, shows every shift in the data as a trend.

Donovan (1993) shows results of reliability growth testing of the infrared camera on the B-52 bomber. The chart of actual results shows an increase in MTBF in each time period for nine successive time periods. Without statistical confidence lines drawn, one would conclude that this is a legitimate trend of reliability growth. If there were less time periods of subsequent growth (ie. five or less), then no conclusion could have been drawn without statistical confidence lines being drawn.

Jokubaitus et al. (1992) presents an approach for planning a reliability and maintainability program which takes into account reliability growth through Crow's AMSAA growth testing techniques. The paper furthers the discussion into the field to assess the viability of Reliability improvements for fielded systems. The authors allude to techniques used by the US Census Bureau to improve the accuracy of collected data. The authors advance the concept of collecting growth data in the field. Reliability Growth models can be applied to accurate field data to assess the change in reliability in the field if it can be established that common causes are acting on the system, not special causes.

Ellis (1992) compares Duane model, AMSAA model and Kalman Filtering for ability to detect an exponential distribution without reliability growth. In order to do this, she generated test values from an exponential distribution with an MTBF of 60 hours. After verifying that the test values represent an exponential distribution using a Kolmogorov-Smirnov test, ten sets of test values were tested for reliability growth using all three models. The author found that there was a great deal of subjectivity incorporated in the Duane model causing it to indicate growth when none existed. This phenomenon is especially prevalent when data contains outliers especially in the beginning data. The AMSAA model predicted a stable model well and also gave reliability confidence for its results. The Kalman filter only gives an estimate of reliability at the last value in the model. However, the author states that "if Crow-AMSAA does not fit the data, a (Kalman Filter) technique might be more appropriate". The author found that the methods used for dynamic assessment of a process which are the Duane model and the AMSAA model are inadequate. The Duane model was found to be subjective and easily altered by outliers. The AMSAA model often does not fit the data and does not give a variation analysis for changing processes. The Kalman filter is a point estimate and does not provide dynamic assessment of reliability change with time.

O'Conner (1991) describes the Duane method of reliability growth measurement. The Duane method is based on the work of J. T. Duane, "who derived an empirical relationship based upon observation of the MTBF improvement of a range of items used on aircraft. Duane observed that the cumulative MTBF, θ_c, (total time divided by total failures) plotted against total time on log-log paper gave a straight line. The slope (α) indicates reliability (MTBF) growth, i.e.

(1) $$\log\theta_c = \log\theta_0 + \alpha(\log T - \log T_0)$$

Duane observed reliability growth rates ranging between 0.2 and 0.4 in programs actively pursuing reliability growth.

O'Conner points out that the chief criticism for the Duane model is the fact that it is empirical and therefore subject to wide variation and interpretation. It is also argued that reliability improvement in production is not an incremental function but a step function.

O'Conner discusses reliability growth in service with the following observations:

1) Failure data is difficult to maintain in service. Investigation is difficult when users own the equipment.

2) It is more difficult to modify delivered equipment or to make changes once production has been started.

3) Reliance on reliability growth is expensive in terms of warranty, reputation and markets.

Dovich (1990) outlines the procedures for performing a Sequential Life test to assess MTBF values. Method proposed shows whether the system under test is meeting or failing to meet reliability parameters within given statistical confidence. Given the type of distribution, the null hypothesis (MTBF predicted), alternative hypothesis (unacceptable MTBF), producer's risk, and consumer's risk; a test can be constructed to demonstrate whether a system: a) meets or exceeds specifications, b) fails to meet specifications, or c) needs to be tested further for adequate statistical certainty. Although this method sequentially tests subsequent failures, it does not impart information about the reliability history of the part or system in question. Basically, it is a reactive pass/fail test. The methods for testing both the exponential and the Weibull distribution are presented.

Heimann et al. (1992) addresses a process related statistical method which takes Crow's AMSAA Reliability Intensity function a step further. The Crow model uses the equation:

(2) $$h(t) = \frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta - 1}$$

Where α is the scale parameter and β is the shape parameter which governs the change in failure rate over time. If $\beta < 1$, the process is in the infant mortality stage. If $\beta = 1$, then the process is in the constant failure rate phase. If $\beta > 1$, then the process is in the wearout phase. Reliability growth is usually monitored when in the infant mortality stage.

Although the Crow model is helpful, the authors point out that it could be expanded. This contention stems from the observation that even in the production phase products have several periods of different rates of reliability growth. In order to solve for this observed phenomenon, the authors introduce a function to replace the scale factor α in the production phase:

(3) $$\alpha(T) = \alpha(1 - e^{-bT})$$

Where T is the age of the process and b is a shape parameter. This process is useful in analyzing the dynamics of production. The process takes into account discrete changes in the production process at discrete points in time, and thus cannot take advantage of the superposition principle advanced in Crow 's paper. As such, it does not provide specific information regarding the value of β for the infant mortality phase.

The paper does not give dynamic information on the changes in experienced reliability. The method advanced in the paper also requires a great deal of statistical refinement to attain a mathematical model for a part or system disregarding the fact that many changes are due to simple statistical variation in the data.

Meth (1992) provides a review of Reliability Growth model literature. He points out that the first and most often referenced Reliability model is Duane's

Reliability Growth model which was not intended as a prescriptive model of the reliability of a system but rather as a descriptive analysis of an observed learning curve. Meth sums up the article by stating the opinion that the only reasonable uses for reliability learning curve application are to determine approximate test time requirements and to monitor the rate of reliability in test. Meth states that current reliability growth techniques should not be used to predict or to estimate system reliability. This is because the current methods used for reliability growth studies are not designed to estimate system reliability. Finally, Meth suggests that, in selecting a reliability growth model, one should use the simplest reliability growth model possible and then validate the accuracy of the model before using.

Manipulation of Field Data

Luke et al. (1993a) describes a non-parametric method to construct time estimation when limited field time meter readings are available. The method espoused by the paper is to take existing time meter readings and reduce to percentage of calendar time the system is stressed. Percentage calendar time is then tested for accuracy of a median estimate using the Wilcoxon test. Once a good estimate of the median is obtained, it is used to estimate stress time for all ships which have no reported time meter readings. Ships with time meter readings use actual data for test results. This method is a good one to estimate time when data is lacking since it relies on actual data and is checked for accuracy against existing data.

Determining Distribution Parameters

Giuntini et al. (1993) illustrates a technique for combining predicted (*a priori*) reliability distribution and limited field data to produce a posterior distribution. The method is simply to start with a proposed (*a priori*) distribution for reliability and combine the proposed distribution with a distribution derived from field data to obtain a new proposed (posterior) distribution. The posterior distribution is then used as the *a priori* distribution for the next iteration with new field data. The process is repeated until the analyst feels comfortable with the estimate.

Although the article is written for a Weibull distribution, it will also work for an exponential distribution because the exponential distribution is a special case of the Weibull ($\eta = 1$, and $\beta = \lambda$). The limitation of this paper is that it assumes that field reliability is static. If the data is static, then the posterior distribution should eventually stabilize. If field reliability is changing over time, the results will favor past data over present data. Although the posterior estimate will change if field data changes, there is not a way to distinguish between changes due to special causes and changes due to common causes. With the accumulation of time, the posterior becomes more insensitive to process changes.

Statistical Process Control for Reliability

Hugge (1993) suggests a quality assurance approach be adopted for reliability programs. Hugge emphasizes the need to know whether reliability performance is continuously improving. Hugge suggests that a six sigma approach to center the process be employed. He does not specify what parameter would be measured or what distribution would be used. It is assumed that he is measuring time to failure using SPC techniques.

Hugge suggests using a six sigma approach to monitor major failure modes which are discovered by categorizing failures and placing them in a Pareto diagram. Although this will work in theory, there is usually not enough data in practice to attempt to draw an SPC chart on each failure mode. In order to establish control, one needs approximately 100 data points. In Yang's (1993) work, there were only six failures over a 24 month period from all failure modes. In Perera's (1993) work, there were only 19 failures over a one year period from four failure modes, ten of which were from the primary failure mode. In order to use SPC, so many years of data would be required and the systems would probably reach obsolescence first. The tenet of Hugge is that reliability parameters should be analyzed dynamically and statistically so that continuous improvement can be documented and variation can be controlled.

Conclusion

Currently, there is not a method which can demonstrate changes in reliability of a system or part within a small set of data. There needs to be a method which measures precisely the actual reliability of a system or part and detects when a perturbation to that system causes a change to reliability. The method must be able to give results with statistical confidences so that analysts can decide what degree of confidence the results can be given.

CHAPTER 3.

RESEARCH METHODOLOGY

Research Model

The proposed approach, LRTC, is based on the assumption that an electronic system's reliability can be modelled as exponentially distributed function. Reliability confidence intervals for the exponential distribution can be estimated using a χ^2 distribution (Dovich 1990). If a test is truncated at a predetermined number of failures, reliability is estimated as being between a lower confidence limit (LCL) and an upper confidence limit (UCL) according to the equations:

$$LCL = \frac{2T}{\chi^2_{(\frac{\alpha}{2},2r)}} \tag{4}$$

$$UCL = \frac{2T}{\chi^2_{(1-\frac{\alpha}{2},2r)}} \tag{5}$$

$$LCL \le MTBF \le UCL \tag{6}$$

where T is total Time, α is the confidence level and r is the number of failures.

If a test is truncated at a predetermined time, reliability is estimated according to the equations:

(7) $$LCL = \frac{2T}{\chi^2_{(\frac{\alpha}{2},2r+2)}}$$

(8) $$UCL = \frac{2T}{\chi^2_{(1-\frac{\alpha}{2},2r+2)}}$$

SPC rules analyze patterns of normally distributed data to detect out of control processes. The rules are based on the location of data points relative to the mean of the distribution. The reasoning behind these patterns is built on the known probabilities of a point lying beyond one, two, or three standard deviations from the mean (Gitlow et al. 1991, Luke et al. 1993b). The rules for detecting that a process is out of control are:

1) One value over $\pm$ 3 σ.

2) Two of three consecutive values over $\pm$ 2 σ.

3) Four of five values on the same side of center over $\pm$ 1 σ.

4) Six values in a row increasing or decreasing.

5) Eight consecutive values, none of which are within $\pm$ 1 σ.

6) Nine values in a row, all on the same side of the centerline.

7) Fourteen values in a row alternating up and down.

8) Fifteen values in a row between $\pm$ 1 σ.

These rules are based on known probabilities of the normal distribution. The reliability of a system is estimated using the χ^2 distribution making these rules inappropriate for reliability. However, similar rules could be developed if values with the same probabilities as 3, 2, 1, 0, -1, -2, and -3 σ in the normal σ's could be established. For convenience, these values are referred to as being 1z, 2z, 3z, 0z, -1z, -2z, -3z referring to the z-statistic value for the normal distribution with a mean of 0 and a standard deviation of 1. Rules 7 and 8 are not considered important in Reliability studies. Rule 7 is concerned with overadjustment which is not a factor in reliability. Rule 8 is concerned with processes whose standard deviation has reduced while the mean has remained constant, which does not make sense for an exponential distribution where the mean and the standard deviation are the same number (MTBF).

There is an equation which expresses the χ^2 distribution in terms of the z-statistic (Ireson et al. 1988). The LRTC uses this equation to establish values at -3z, -2z, -1z, 0z, 1z, 2z, and 3z for a reliability function over a period of time. Once the transformation is made, then the same rules as were used for SPC pattern analysis can be used to analyze reliability trends. The equation is:

(9) $$\chi^2(\alpha,x) \approx x(1 - \frac{2}{9x} + z_{\alpha}\sqrt{(\frac{2}{9x})})^3 \text{ (\textit{Ireson et. al.}, 1988)}$$

In order to use this equation, the following steps are taken:

1) Establish the MTBF for the Process in question:

(10) $$MTBF=\frac{N}{T}$$

2) Determine whether it is more convenient to divide the process into time-truncated period or failure truncated periods.

3) Determine the expected number of failures for each period:

(11) $$r=\frac{MTBF}{t}$$

4) Calculate the values for lines drawn at $2T/\chi^2(\alpha,x)$ where $z_\alpha = 3, 2, 1, 0, -1, -2, -3$ and $x = 2r$ for failure truncated periods and $x = 2r+2$ for time truncated periods.

The equation for χ^2 must be used instead of standard χ^2 tables because the values for n may not be whole numbers or may exceed the limitations of the chart. The value of MTBF is not the value for $z_\alpha = 0$ because the latter is an estimate of the median and the former is an estimate of the mean. In the exponential distribution, the median and the mean are not coincident.

For further clarity and to reduce distortion, the LRTC is normalized using the following equation for the z-statistic:

(12) $$z=\frac{\sqrt[3]{\frac{2T}{TBF(x)}}-1+\frac{2}{9(x)}}{\sqrt{\frac{2}{9(x)}}}$$

Where x = 2r for failure truncated periods and x = 2r+2 for time truncated periods.

If the z statistic is plotted, then the reliability lines are all straight and equidistant, making analysis of trends much easier. The normalized chart is used to analyze trends and determine MTBF for periods in which the process is within control.

Design of Experiments

An actual case study may contain unknown and uncontrollable sources of variation which can confound results of analysis. In order to preclude this problem, a simulation with a known exponential distribution and a known MTBF of 400 hours was used to test the LRTC. This distribution was constructed by setting the cumulative density function of the exponential distribution equal to a random number between zero and one. 100 values for the time to failure, t, are then generated using the equation:

(13) $$t = -400\ln(1 - RAND)$$

The LRTC must also be tested for processes under change. To accomplish this, a second simulation is established which has 50 points from a distribution with MTBF of 400 hours and 50 points with an MTBF of 200 hours:

(14) $$\begin{aligned} t_1 \cdots t_{50} &= -400\ln(1 - RAND) \\ t_{51} \cdots t_{100} &= -200\ln(1 - RAND) \end{aligned}$$

In order to perform analysis with the LRTC, the following assumptions are made:

1) Electronic systems' reliability is exponentially distributed.
2) χ^2 confidence intervals are appropriate to estimate reliability.
3) Enough failures have occurred for time truncated estimation to be relatively unbiased.
4) χ^2 equation is an appropriate approximation of the χ^2 distribution.
5) Conclusions drawn for systems can be applied to lower levels such as units, parts, and components.
6) Inferences drawn from shifts with a discrimination ratio of two are applicable to systems with larger and smaller discrimination ratios.
7) Downward shifts in MTBF act essentially identical to upwards shifts in MTBF.

8) Equipment is in "useful life phase" where reliability is constant, not in infant mortality phase where reliability is steadily increasing or in wearout phase where reliability is steadily decreasing.

9) Reliability changes in the useful life phase can be modelled as step functions.

The simulation experiments are designed to compare the LRTC to the Sequential Life test, the Duane Reliability Growth model, the Posterior test, and SPC. The random number generator used was the function "RAND()" in Excel 4.0 which produces a random value between 0 and 1.

Test of Hypotheses

Hypothesis $H1_1$

In order to prove that the LRTC describes systems with an exponential reliability distribution, a set of failure data is generated with a known exponentially distributed hazard function distribution. The LRTC is tested to observe the following:

1) Mean squared error (MSE) of model estimate of MTBF versus true MTBF:

$$MSE = \sum_{i=1}^{m} (MTBF_i - 400)^2 \tag{15}$$

2) Ability of model to detect that systems with a constant MTBF are "in control" (avoid a Type I error). This is analyzed in more detail in hypotheses $H0_2$ through $H0_5$.

3) Ability to detect a shift in process when a shift occurs (avoid a Type II error). This is analyzed in more detail in hypotheses $H0_6$ through $H0_9$.

Hypothesis $H0_2$

The LRTC is compared to the Sequential Life test for ability to avoid a Type I error by running multiple simulations of random generated models of a system with 100 failures and an MTBF of 400 hours. For the Sequential Life test model, each model is categorized as "accept" or "null", or "reject" or "Type I error". The Sequential Life test was set up with a discrimination ratio of 4, a producer's risk of 0.10 and a consumer's risk of 0.05. For the LRTC, the random generated values are tested for "out of control" conditions if any of the following patterns develop:

1) One value over ± 3 z.

2) Two of three consecutive values over ± 2 z.

3) Four of five values on the same side of center over ± 1 z.

4) Six values in a row increasing or decreasing.

5) Eight consecutive values, none of which are within ± 1 z.

6) Nine values in a row, all on the same side of the centerline.

The LRTC models are categorized for "in control" or "null" and "out of control" or "Type I error". Both the LRTC and the Sequential Life test are then evaluated using a χ^2 test with three degrees of freedom and a 97.5% statistical confidence level.

Hypothesis $H0_3$

The Duane Reliability Growth model was categorized as "null" if the value or R^2 for the best fit regression line of each set of 100 data points is less than 0.64. In such an instance, the estimate for MTBF is the average of all the Times Between Failure for the 100 values.

The Duane Reliability Growth model is categorized as "Type I error" if the value of R^2 is greater than 0.64, implying that a linear relationship between time and MTBF exists. In such an instance, MTBF is estimated as the result of the regression line for the last point of data.

The results from the analysis of the Duane Reliability Growth model and the LRTC are placed in a χ^2 test with 3 degrees of freedom and a 97.5% statistical confidence level. The LRTC error is calculated by taking the average of the values available for analysis. The MSE of 132 test values of the LRTC are compared to the average of 132 MSE test values from the Duane Reliability Growth model.

Hypothesis $H0_4$

The Posterior test was categorized as "null" if no value for the upper limit at 90% confidence was below the actual MTBF of 400 hours and no value for the lower limit at 90% confidence was above 400 hours MTBF.

The Posterior test was categorized as "Type I error" if any value for the upper limit at 90% confidence was below the actual MTBF of 400 hours or if any value for the lower limit at 90% confidence was above 400 hours MTBF. The error for the Posterior test is the difference between the terminal mean estimate of MTBF and 400 hours.

The results from the analysis of the Posterior test and the LRTC are placed in a χ^2 test with 3 degrees of freedom and a 97.5% statistical confidence level. The LRTC is calculated by taking the average of the values available for analysis. The MSE of 132 test runs of the LRTC is compared to the average of 132 MSE test values from the Posterior test.

Hypothesis $H0_5$

For SPC, 20 subgroups of size 5 are chosen in the experiment. An X-Bar and an R-chart are then drawn and analyzed for patterns. For the SPC Chart, the random generated values are tested for "out of control" conditions using the SPC rules described in the Research Model Section.

The SPC Chart models are each categorized for "in control" or "null" and "out of control" or "Type I error". For the LRTC, the random generated values are tested for "out of control" conditions using the same conditions as in Hypothesis $H0_2$. Both the LRTC and the SPC test are evaluated using a χ^2 test with three degrees of freedom and a 97.5% statistical confidence level.

Hypotheses $H0_6$ through $H0_9$

A random model is used in a similar fashion as $H0_2$ through $H0_5$ except the process shifts from 400 hours to 200 hours MTBF at point 51. If each respective model fails to recognize the shift, it is categorized as "Type II Error". When each model recognizes respective shift, it is categorized as "null". All models are tested against the LRTC using a χ^2 test with 3 degrees of freedom and a statistical confidence of 99.7%.

CHAPTER 4

ANALYSIS AND DISCUSSION OF EXPERIMENTS

Introduction

In order to test the proposed model versus established methods of detecting reliability and changes in reliability, a simulation was devised. In the simulation, a set of data with a known exponential distribution was tested to observe whether each of several detection techniques could demonstrate whether the process was stable or had changed with time. If the process was found to be stable, the test case was further examined for whether or not it could predict the true starting and finishing reliabilities, expressed in terms of mean time between failures (MTBF) for a system. The control case simulation chosen was for a system with MTBF of 400 hours. The simulation was performed for two cases: a) one where reliability was kept constant, and b) one where reliability shifted with time in a known fashion.

After the simulations were complete, the LRTC technique was applied to three case studies. The three studies chosen were all power supply examples, chosen from actual applications. They were chosen because they represent actual problems in the fleet. They are equipment on government systems and are referred to as "Power Supply A", "Power Supply B", and "Power Supply C" in this research. The results from the analyses of the LRTC charts drawn for these three power supplies have shaped government decision making and have produced savings of over $900,000.

Random Number Generated Simulation

In reliability engineering, if the components of a system all have constant failure rates, then the failure rate of the system is the sum of the failure rates of the components. If there is variation present, each component may or may not achieve its own predicted failure rate. Usually, the system will come close because the individual variations of the components tend to cancel each other out. The failure rate of a system, expressed in units failures per million hours, fpmh, can be extremely large, so system reliability is usually expressed in terms of the inverse of failure rate, MTBF with units of hours.

In this study, system failure rate is first analyzed for a test case with known MTBF of 400 hours. Next the test case is forced to shift from 400 hours to 200 hours at point 51. A simulated distribution with 400 hours MTBF was generated with a random number generator. As stated earlier, an equation (13) is used to ensure that the prescribed followed an exponential distribution with a MTBF of 400 hours. At first, a complete set of graphs with accompanying analyses is performed on one simulation for all models. Then, each method is simulated multiple times to compare statistically the results of the LRTC and the other models.

Simulation, Constant Mean Time Between Failures

First, one set of values for the constant 400 Hr MTBF system case were generated using equation 13. The random number for equation 13 was generated on a TI-81 calculator and transferred onto a spreadsheet. Then, the values were used using various analysis techniques to observe whether each technique would adequately predict that the process is not changing and has an average MTBF of 400 hours. The failure times obtained are shown in the Appendix.

LRTC Methodology

The LRTC was drawn for the 400 Hour MTBF process with failure-truncated periods of five failures each. Reliability distributions of electronic systems are assumed to be exponential in nature, therefore reliability confidence can be estimated with the χ^2 distribution. Since the periods are failure truncated, the values for the UCL and LCL are obtained from equations (4) and (5). Similarly, the seven contour lines for the LRTC are also obtained with 2n degrees of freedom for the χ^2 estimator of the number of failures at the various confidence levels. To draw the LRTC, reliability contour lines at each time period are drawn according to the following equations:

(16) $$Mean = \frac{N}{T}$$

(17) $$\chi^2(\alpha,x) \approx x(1 - \frac{2}{9x} + z_\alpha\sqrt{(\frac{2}{9x})})^3$$

$$MTBF\,at\,confidence\,\alpha = \frac{2T}{\chi^2(\alpha,2n)}$$

(18)
$$\approx \frac{2T}{2n(1-\frac{1}{9n}+z_\alpha\sqrt{\frac{1}{9n}})^3}$$

$$\approx \frac{T}{n(1-\frac{1}{9n}+z_\alpha\sqrt{\frac{1}{9n}})^3}$$

(19)
$$n = \frac{t}{mean}$$

The equation for χ^2 must be used instead of standard χ^2 tables because the values for n may not be whole numbers or may exceed the limitations of the table. For the confidence levels stipulated, the values for z_α are -3, -2, -1, 0, 1, 2, and 3. The median is where z_α equals zero. The value for the mean which is the estimated MTBF is not drawn on the chart.

The standard rules for SPC are then applied on the resultant chart. For figure 1, the process appears to be in control. For further clarity and to reduce distortion, the LRTC is normalized using equation (12) as discussed earlier.

Figure 2 is the Normalized LRTC. The LRTC estimated average MTBF at 372.5 hours. The chart shows no particular trends or changes in the process.

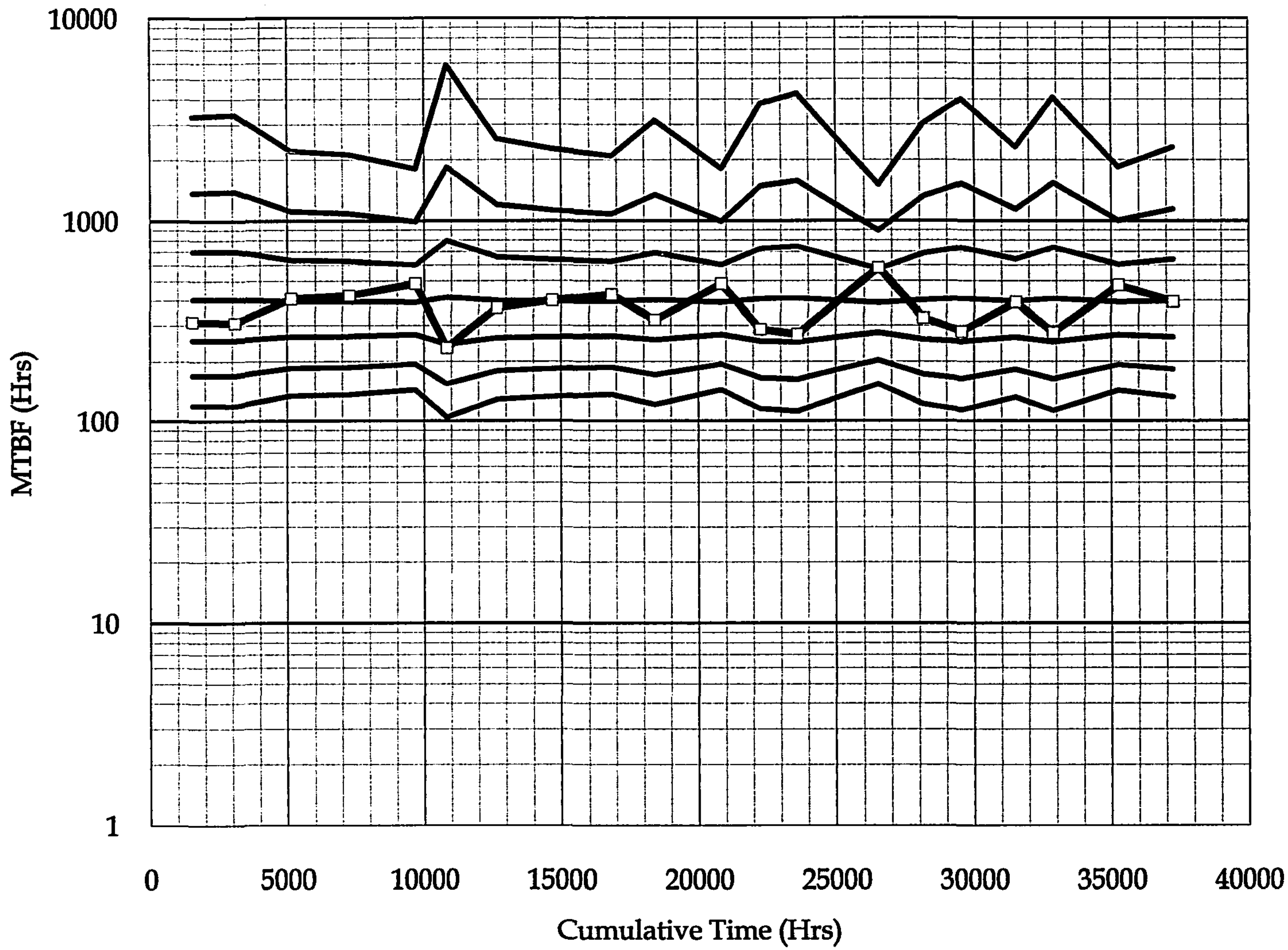

Figure 1 . LRTC 400 Hour Simulation

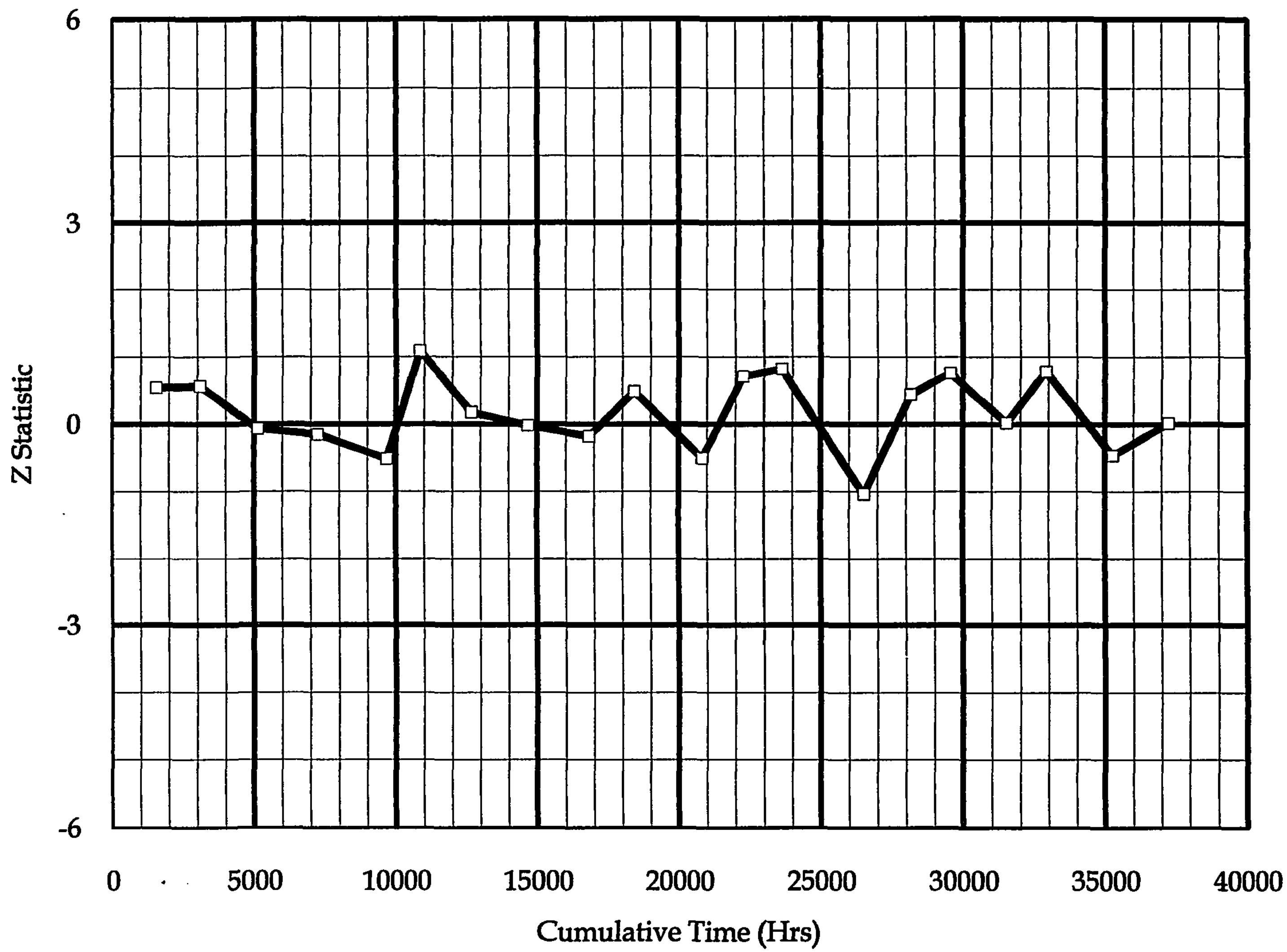

Figure 2. Normalized LRTC for 400 Hour Simulation

Test for Ability to Detect the Mean, Hypothesis $H1_1$

The LRTC was compared to the Duane Reliability Growth model, the Posterior test and SPC for the estimation of MTBF. The Sequential Life test method does not give an estimate for the mean and thus could not be compared to the rest of the methods. For the methods compared, the MSE was used as the basis of the comparison (table 1).

TABLE 1

MSE TEST FOR $H1_1$

Method	MSE
LRTC	1445.4
Duane Reliability Growth Model	2980.2
Posterior Test	1445.4
SPC	1832.0

The LRTC is superior to all the methods except the Posterior test. The Posterior test can only estimate the mean when there is a stable process. The Posterior test is insensitive to processes undergoing change as is demonstrated in the analysis of Hypothesis $H0_8$. Therefore, the LRTC is the best method for estimating the mean for processes under change.

Sequential Life Testing

Sequential Life testing was applied to see if it could be used to verify that the MTBF of the process is 400 hours. In order to perform a Sequential Life test, the following are established:

θ_0, acceptable MTBF = 400 hours.

θ_1, unacceptable MTBF = 100 hours.

α, producer's risk = 0.05 (This is a typical number used in industry. It means that there is a 95% statistical certainty that a process which is really 400 hours will not be rejected.)

β, consumer's risk = 0.10 (This is a typical number used in industry. It means that there is a 90% statistical certainty that a process which is really 100 hours will not be accepted.)

In order to analyze a Sequential Life test, failure number, n, is plotted as the independent variable and cumulative hours, T, as the dependent variable. Two lines on the graph divide the area into three regions: 1) accept region, 2) reject region, and 3) continue to test region. The equation for the line between the accept region and the continue to test region is:

(20)
$$T = \frac{n\ln(\frac{\theta_0}{\theta_1}) + \ln(\frac{1-\alpha}{\beta})}{\frac{1}{\theta_1} - \frac{1}{\theta_0}} = 184.84n + 300.17$$

The equation for the line between the reject region and the continue test region is:

$$T=\frac{n\ln(\frac{\theta_0}{\theta_1})-\ln(\frac{1-\beta}{\alpha})}{\frac{1}{\theta_1}-\frac{1}{\theta_0}}=184.84n-385.38 \tag{21}$$

Once a test point passes into the accept or the reject region, the test is terminated. Referring to figure 3, after the third failure, sufficient time had passed before the fourth failure to justify acceptance of the sample. Testing would have therefore been terminated at that time (T= 1039 hours) because the fourth failure had not yet occurred.

The Sequential Life test indicates that the system under analysis has a MTBF of at least 400 hours with 95% statistical certainty. The test does not indicate or estimate the actual MTBF and does not indicate whether the system has changed or is changing over time. However, the test is relatively inexpensive since it only required 1039 hours of test time to verify the predicted MTBF.

<u>Comparison of Sequential Life Test to LRTC, Hypothesis $H0_2$</u>

The Sequential Life test was performed on 132 sets of random samples from a failure distribution with an MTBF of 400 hours. In these 132 tests, 7

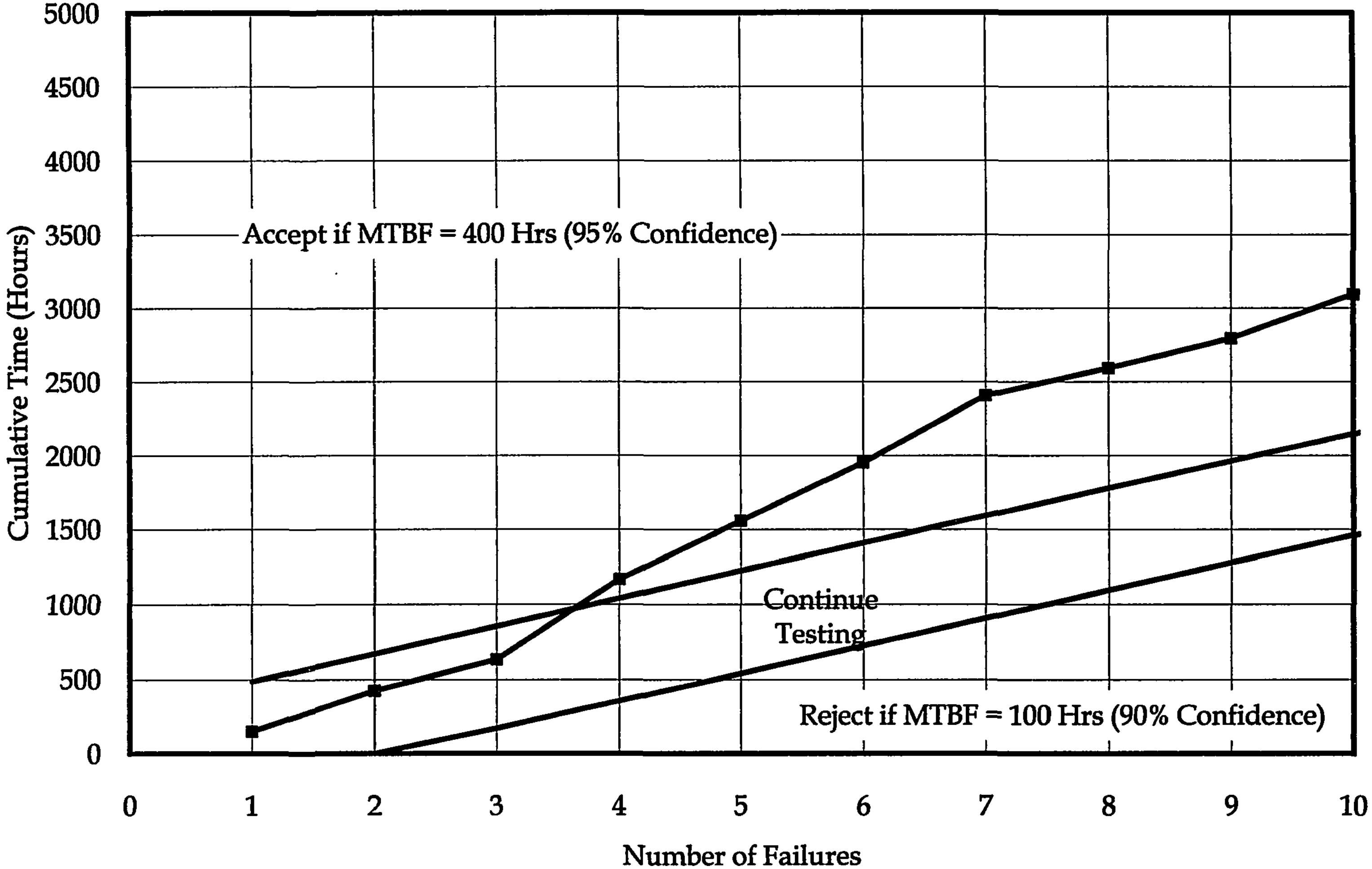

Figure 3. Sequential Life Test for 400 Hour Simulation

ended in rejection using the rules of Sequential Life test and equations (20) and (21). Similarly, the same 132 tests were ran for the LRTC model. Of the 132 tests, 12 were rejected because they appeared to exhibit one of the patterns for an out of control condition (table 2).

TABLE 2

χ^2 TEST FOR $H0_2$

	TYPE I	Null	Total	Q value
SEQ	7	125	132	1.42
LRTC	12	120	132	
Total	19	245	264	

The value of the test is 1.42 which is below the value for the χ^2 with 3 degrees of freedom at 97.5% statistical confidence, therefore the LRTC is not better or worse than the Sequential Life test to detect a process is stable. $H0_2$ is therefore not rejected. From $H1_1$ results, it was established that LRTC is superior to Sequential Life testing in estimation of MTBF.

Duane Model for Reliability Growth Testing

The data from the 400 Hour MTBF distribution was also tested using the Duane Reliability Growth model. In the Duane model, the dependent variable is cumulative time, T, and the independent variable is cumulative MTBF, T ÷ n. The reliability growth factor, α is the slope of a best fit line through the data points plotted on a log-log scale.

Referring to figure 4, there appears to be slight reliability growth. The value of R^2 for the line is 0.6801 and the slope is 0.103. A value of 0.6801 for R^2 indicates that approximately 68% of the data can be accounted for by the line. This is not a strong regressional relationship, but it would probably be reported as reliability growth on a typical reliability growth report even though no reliability growth actually exists. The fact that the Duane Reliability Growth model often indicates growth when growth is not present has been noted in the literature (Demko 1993; Ellis 1992).

The Duane reliability growth model did not provide an accurate estimate of system MTBF. The Duane chart showed growth when none actually existed. The Duane model shows a dynamic relationship of system MTBF over time, but the dynamic relationship does not exist.

<u>Comparison of Duane Reliability Growth Model to LRTC, Hypothesis $H0_3$</u>

The Duane Reliability Growth model was performed on 132 sets of random samples from a failure distribution with an MTBF of 400 hours. In these 132 tests, 34 indicated that there was a reliability trend when there was not a reliability trend. Similarly, the same 132 tests were ran for the LRTC model. Of

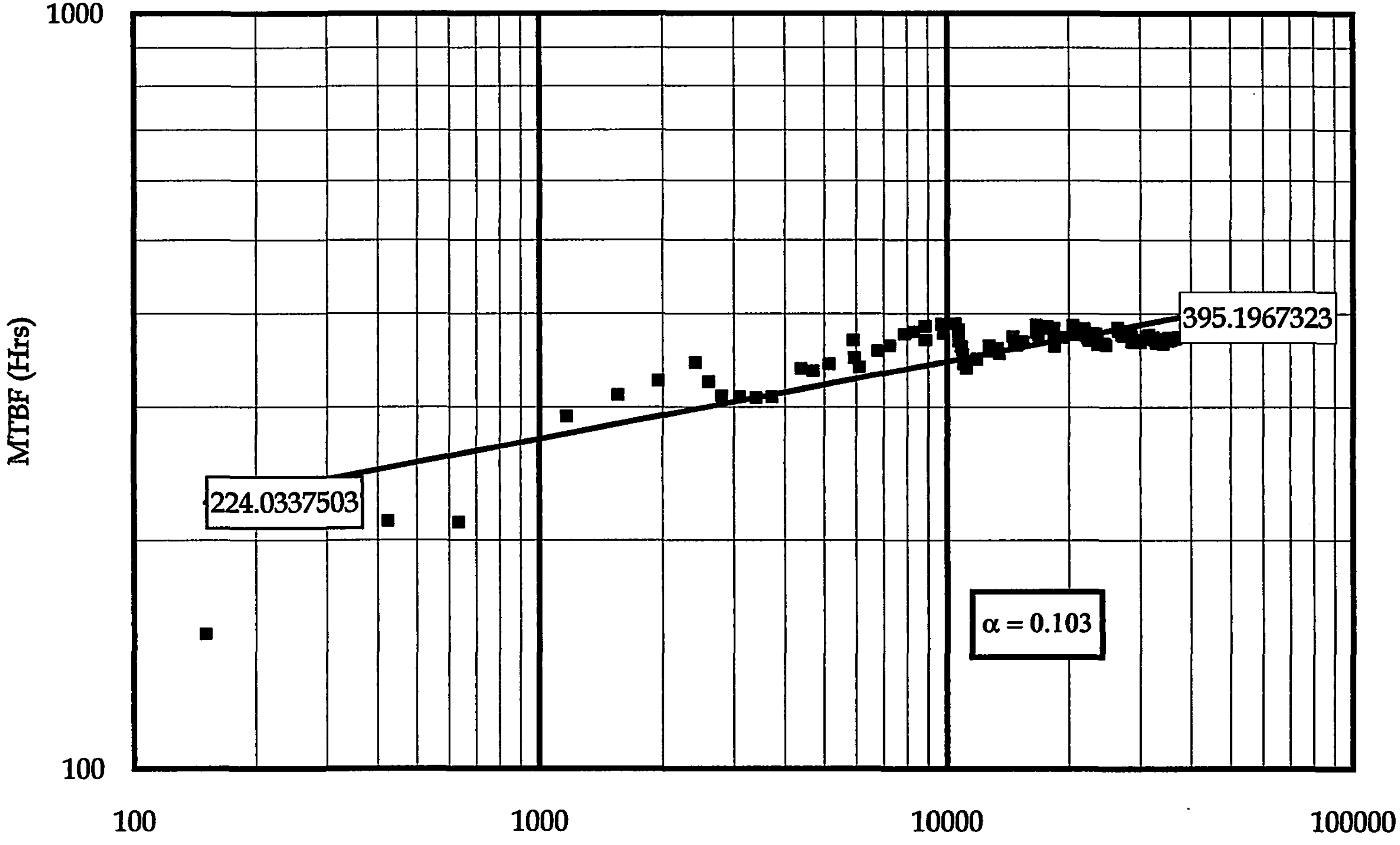

Figure 4 . Duane Model for 400 Hour Simulation

the 132 tests, 12 were rejected because they appeared to exhibit one of the patterns for an out of control condition (table 3).

TABLE 3

χ^2 TEST FOR $H0_3$

	TYPE I	Null	Total	Q value
Duane model	34	98	132	12.74
LRTC	12	120	132	
Total	46	218	264	

The value of the test is 12.74 which is above the value for the χ^2 with 3 degrees of freedom at 97.5% statistical confidence (9.35). Therefore, the LRTC is better than the Duane Reliability Growth model to detect whether a process is stable. $H0_3$ is therefore rejected, implying that there is a difference between the Duane Model and LRTC in avoiding making a Type I error.

From $H1_1$, the Duane Reliability Growth model produced a MSE of 2980.2 Hr^2 for 132 test samples, whereas the LRTC produced a MSE of 1445.4 Hr^2. The LRTC is therefore superior to the Duane model in avoiding Type I error and in estimating the mean of a process.

Posterior Test Using Cumulative Mean Time Between Failures

The posterior or cumulative failure rate test predicts MTBF as the cumulative MTBF. If it is plotted against time, it can be analyzed for drift over time. MIL-HDBK-189 shows that, for an exponential distribution, confidence can be given for a point estimate of failure censored reliability data using a χ^2 distribution according to the equation:

(22) $$\frac{2T}{\chi^2(\frac{\alpha}{2},2n)} \geq MTBF \geq \frac{2T}{\chi^2(1-\frac{\alpha}{2},2n)}$$

where α is one minus the statistical confidence expressed as a fraction.

Referring to figure 5, upper and lower control lines are drawn at statistical confidence of 90%. There is not a change in reliability tested using the cumulative method because the current estimate of reliability never passes through a previous upper or lower limit for reliability. The Posterior test does not detect reliability change and estimates MTBF to be between 300 hours and 450 hours with 90% statistical confidence.

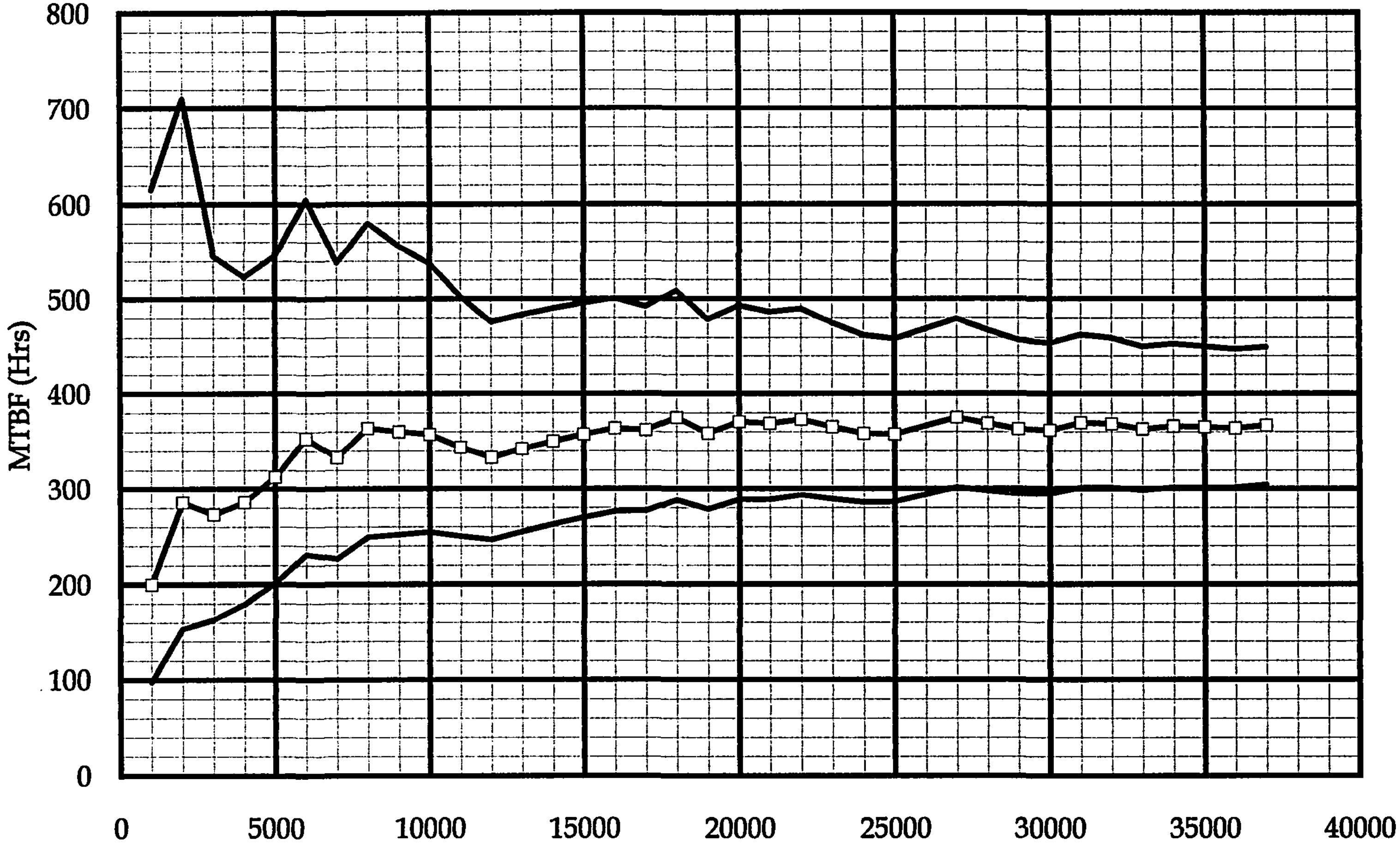

Figure 5 . Posterior Test for 400 Hour Simulation

Comparison of Posterior Test to LRTC, Hypothesis $H0_4$

The Posterior test performed on 132 sets of random samples from a failure distribution with an MTBF of 400 hours. In these 132 tests, 56 indicated that there was a reliability trend when there was not a reliability trend. Similarly, the same 132 tests were ran for the LRTC model. Of the 132 tests, 12 were rejected because they appeared to exhibit one of the patterns for an out of control condition (table 4).

TABLE 4

χ^2 TEST FOR $H0_4$

	TYPE I	Null	Total	Q value
Posterior Test	56	76	132	38.35
LRTC	12	120	132	
Total	68	196	264	

The value of the test is 38.35 which is above the value for the χ^2 with 3 degrees of freedom at 97.5% statistical confidence (9.35), therefore $H0_4$ is rejected, implying that the LRTC is better than the Posterior test to detect whether a process is stable. The Posterior test produced a MSE of 1445.4 Hr^2 for 132 test samples which is identical to the MSE of the LRTC. This because both models compute the mean the same way unless there is a value outside 3z for the

LRTC. The Posterior test is therefore equivalent to the LRTC is estimating the mean but not in avoiding Type I error.

SPC Charts

According to Hogg et al. (1978), the central limit theorem is: "Let X_1, X_2, ..., X_n denote the items of a random sample from a distribution that has mean μ and positive variance σ^2. Then the random variable

$$Y_n = \frac{\sum_{1}^{n} X_i - n\mu}{\sqrt{n}\sigma} = \frac{\sqrt{n}(\bar{X} - \mu)}{\sigma} \tag{23}$$

has a limiting distribution that is normal with mean zero and variance one." According to Grant et al. (1988), "Irrespective of the shape of the distribution of a universe, the distribution of average values, $\bar{T}$'s, of subgroup size n, ($\bar{T}_1$, $\bar{T}_2$, $\bar{T}_3$, . . . $\bar{T}_k$), drawn from that universe will tend toward a normal distribution as the subgroup size n grows without bound." The implication of the central limit theorem is that SPC can be applied to the 100 values for Time To Failure, provided subgroups of the values are used instead of the actual values.

In accordance with common SPC practice, 20 subgroups of size $n=5$ were used. Graphs were then drawn for the averages and the ranges of each subgroup of five values. Referring to the figures 6 and 7, the process appears to be within SPC which means that no special or common cause of variation is apparent. The estimated MTBF of the process is 372.5 hours. This particular SPC chart

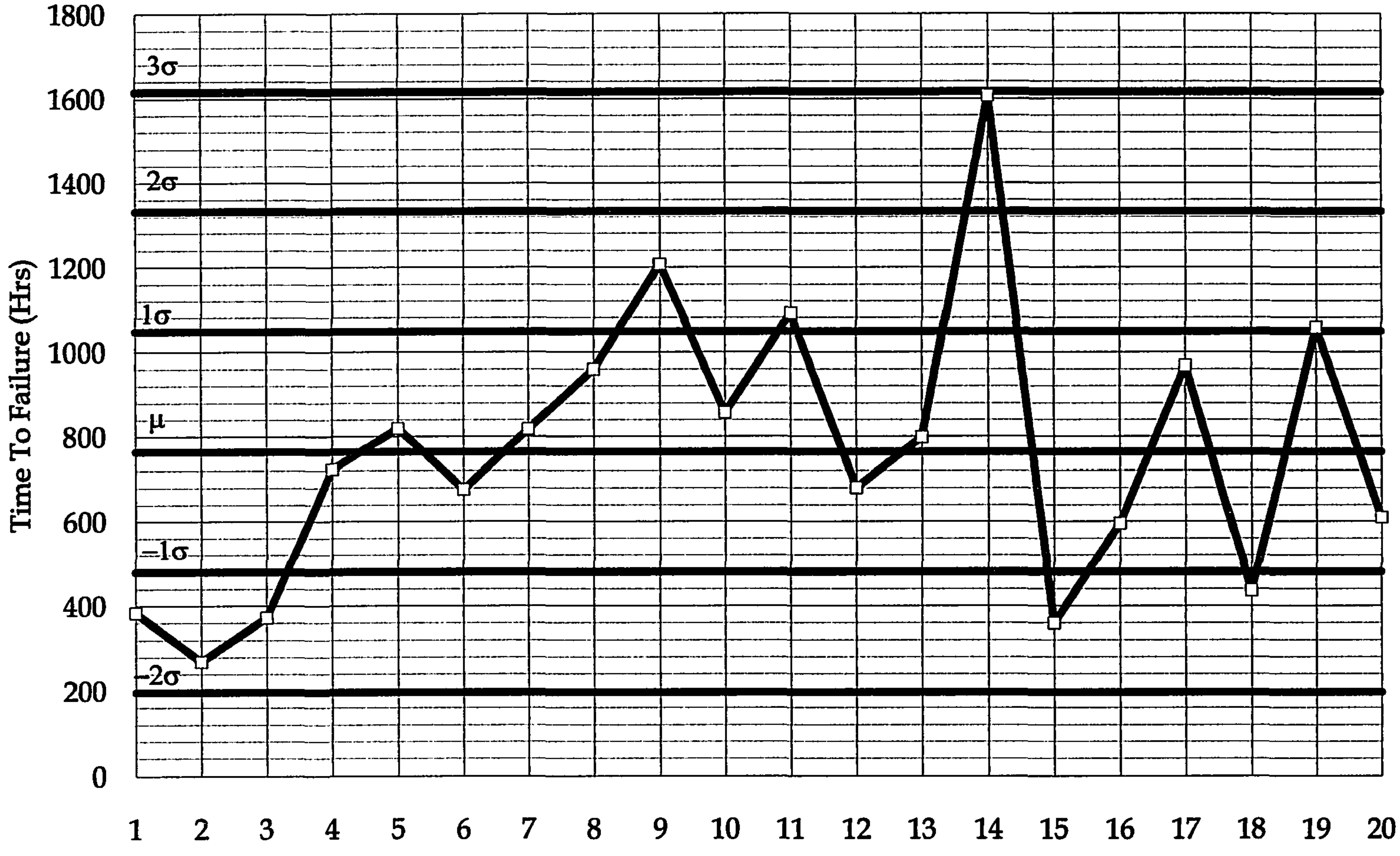

Figure 6. R-Chart for 400 Hour Simulation

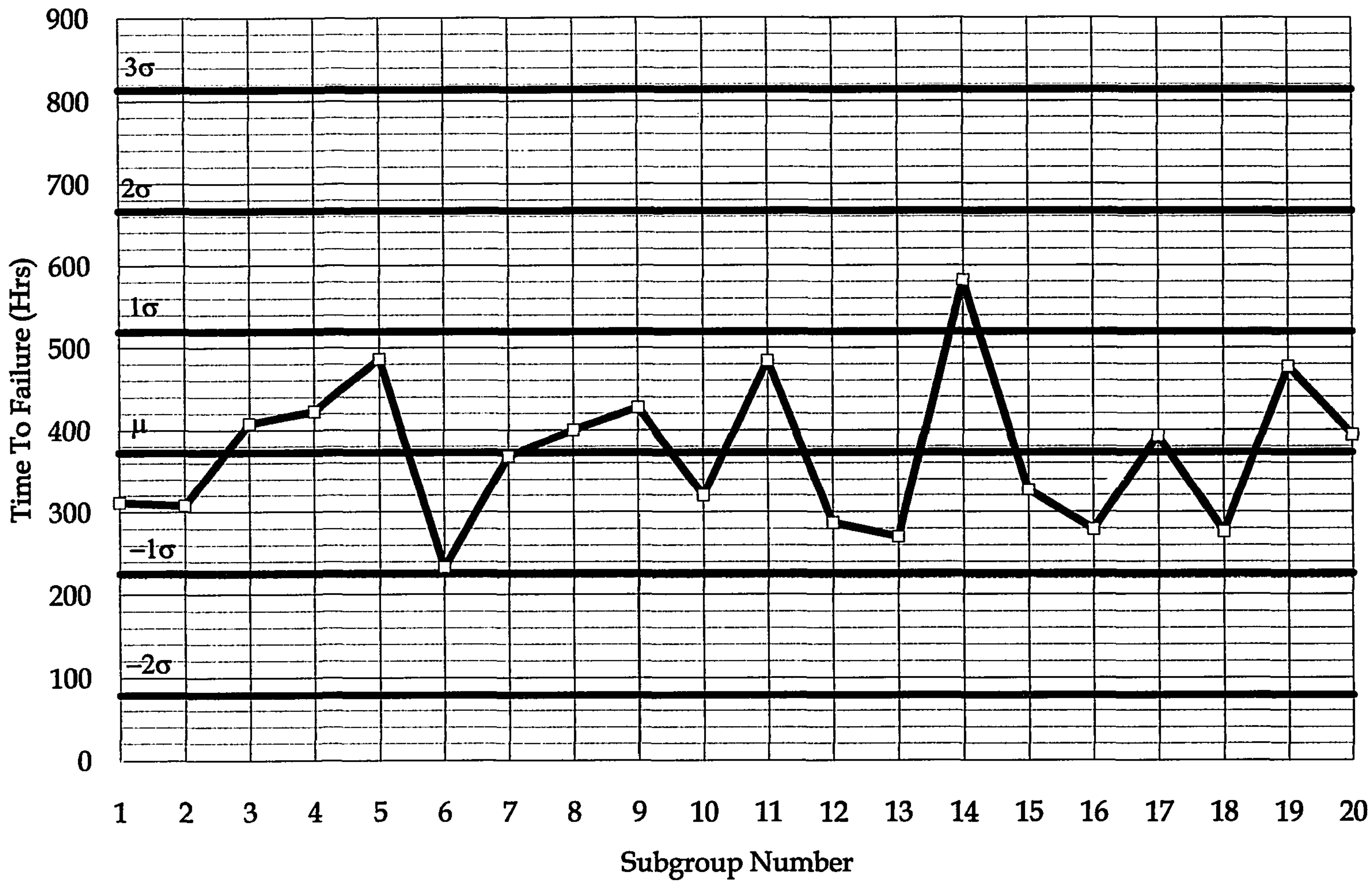

Figure 7 . X-Bar Chart for 400 Hour Simulation

therefore detected that no change to the process had occurred and that the average value is very close to the actual universe value.

Comparison of SPC to LRTC, Hypothesis $H0_5$

The techniques of SPC were applied to 132 sets of random samples from a failure distribution with an MTBF of 400 hours. In these 132 tests, 109 indicated that there was a reliability trend when there was not a reliability trend. Similarly, the same 132 tests were ran for the LRTC model. Of the 213 tests, 12 were rejected because they appeared to exhibit one of the patterns for an out of control condition (table 5).

TABLE 5

χ^2 TEST FOR $H0_5$

	TYPE I	Null	Total	Q value
SPC	109	23	132	143.56
LRTC	12	120	132	
Total	121	143	264	

The value of the test is 143.56 which is above the value for the χ^2 with 3 degrees of freedom at 97.5% statistical confidence (9.35), therefore hypothesis $H0_5$ is rejected. SPC produced a MSE of 1832 Hr^2 for 132 test samples, whereas the LRTC produced a MSE of 1445.4 Hr^2. The LRTC is superior to SPC both in avoiding Type I error and in estimating the mean of a process.

Shift in Process

Although it is important to verify that a process can return a correct response from a system with no reliability degradation or growth, it is a major objective to detect shifts in reliability in real time. In order to test the various reliability assessment methods with a shift in a process, a simulation with a shift in system MTBF from 400 hours to 200 hours was constructed by taking the last 50 points from the distribution described above and dividing the value of t by two. Each of the methods should detect the change and estimate the system MTBF before and after.

<u>LRTC Methodology</u>

Referring to figure 8, the LRTC detects a shift in reliability at time period 11. The shift is detected because for nine periods in a row the reliability of the system was less than median estimate of the MTBF. Because of the distortion due to varying size time periods, the chart is normalized and redrawn in figure 9. The change is interpreted as systemic and the MTBF of the system can be recomputed from periods 1 through 10 and periods 11 through 20.

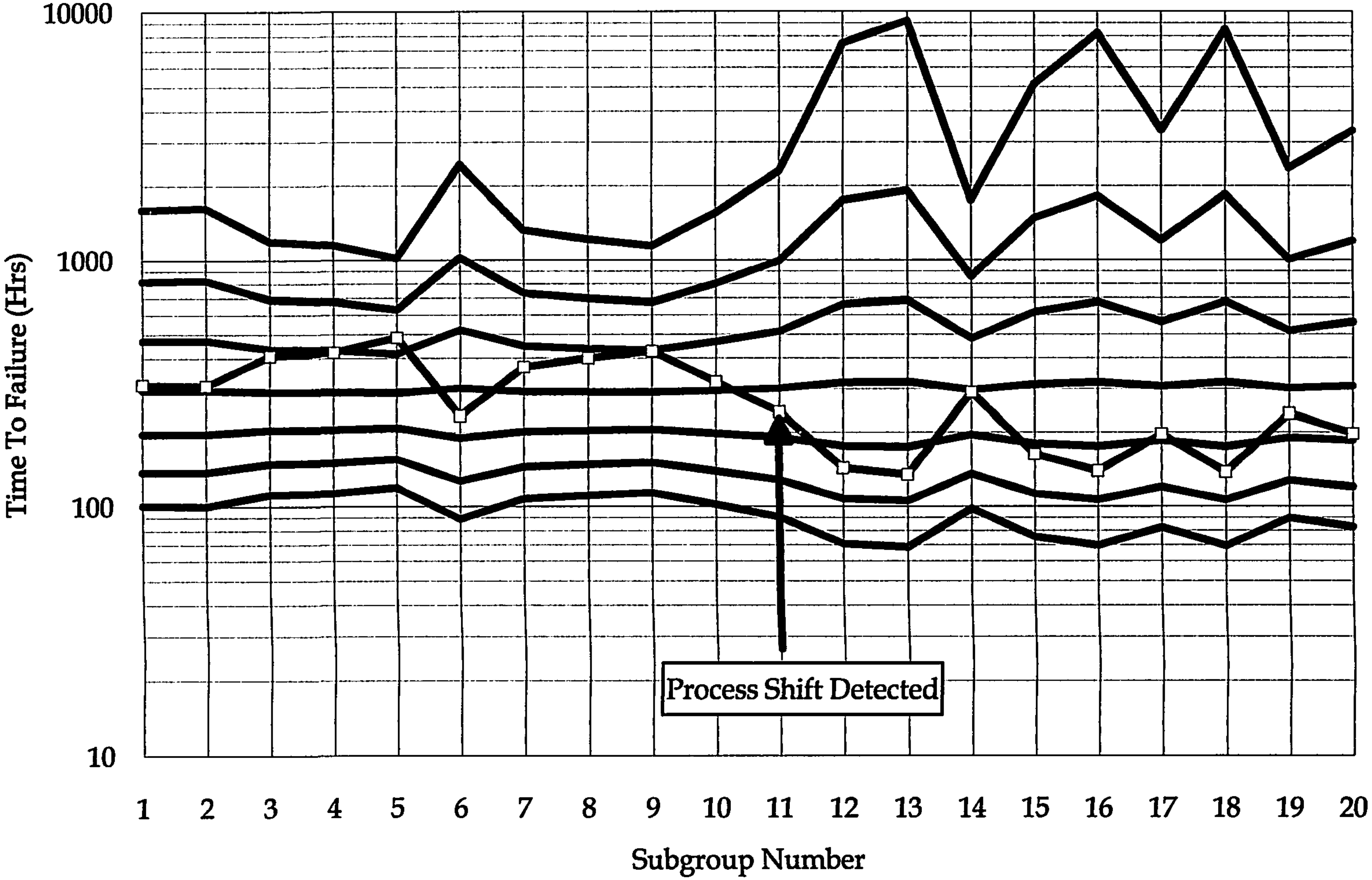

Figure 8. LRTC for 400 Hour Simulation with Shift

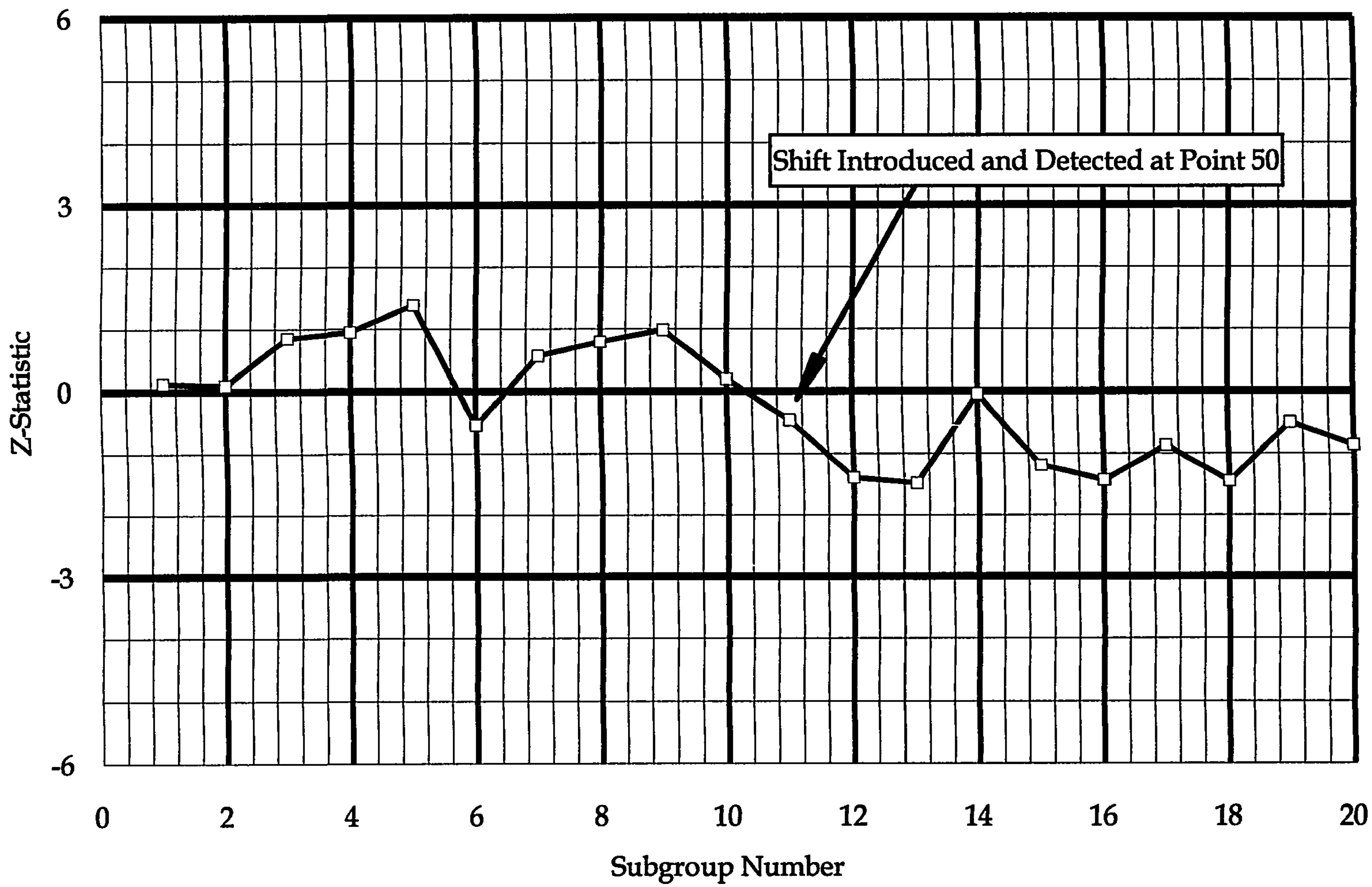

Figure 9. Normalized LRTC for 400 Hour Simulation with Shift

The LRTC is then redrawn to illustrate the change in process (figures 10 and 11). The MTBF for the first 10 subgroups which represent the first 50 failures is estimated at 368 hours. The MTBF for the second ten subgroups which represents failures 51 through 100 is estimated at 188 hours.

Sequential Life Testing

It is assumed that the desired MTBF is the original control MTBF, 400 hours. Applying the same method as for the constant system, figure 12 is obtained. Figure 12 is identical to figure 3 which was drawn for the Sequential Life test of a constant process. This is because the shift in process occurred after the test was already complete. Therefore, between the third and fourth failure it is concluded that the system has an MTBF of 400 hours with 95% statistical confidence.

Comparison of Sequential Life Test to LRTC, Hypothesis $H0_6$

The Sequential Life test was performed on 115 sets of random samples from a failure distribution with an MTBF of 400 hours. In these 115 tests, none were able to detect the change in process because all had terminated the test before the fiftieth failure. Similarly, the same 115 tests were ran for the LRTC model. Of the 115 tests, 34 were rejected because they exhibited a Type II error (table 6).

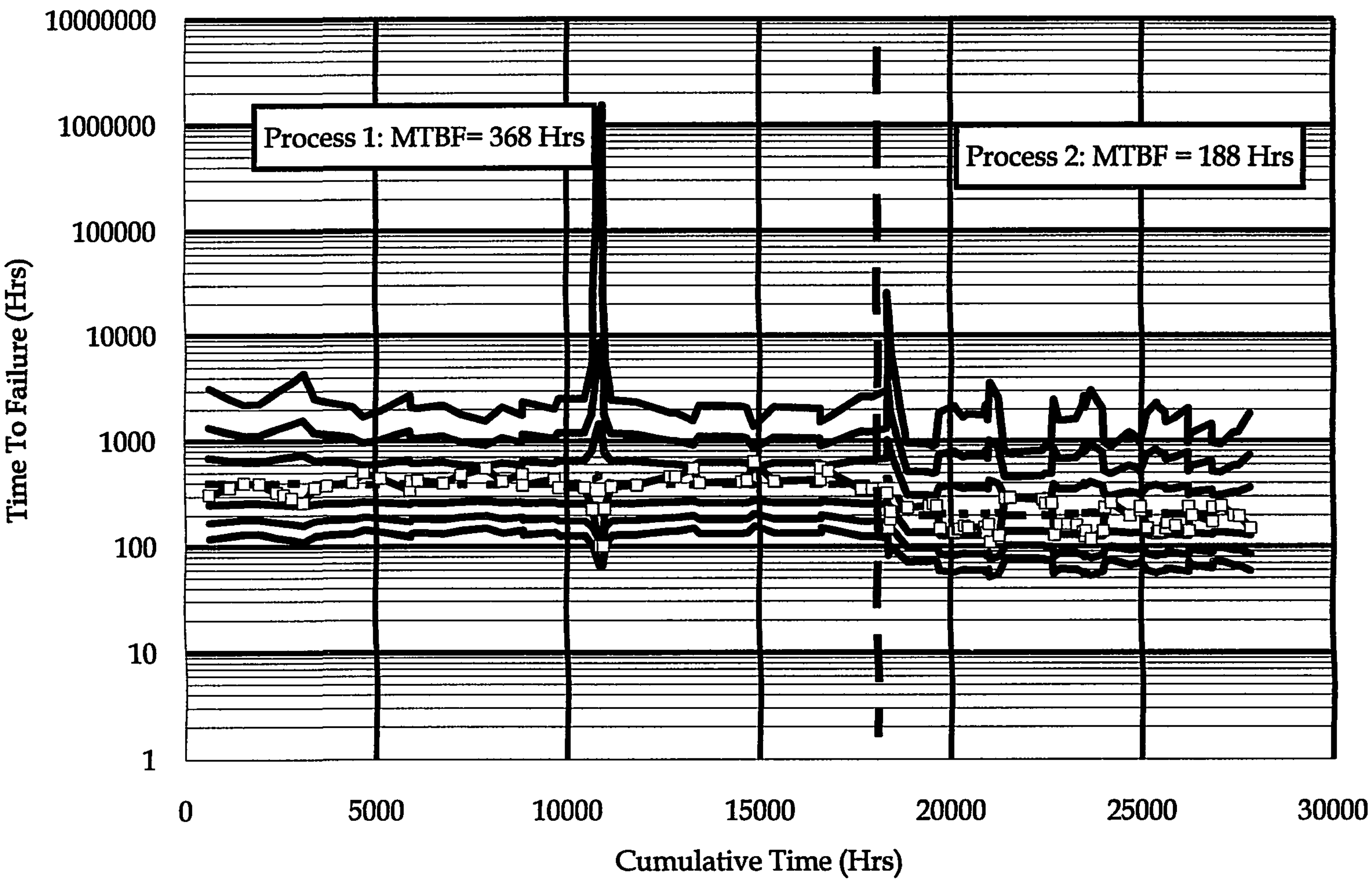

Figure 10. LRTC for 400 Hour Simulation with Shift (Centered)

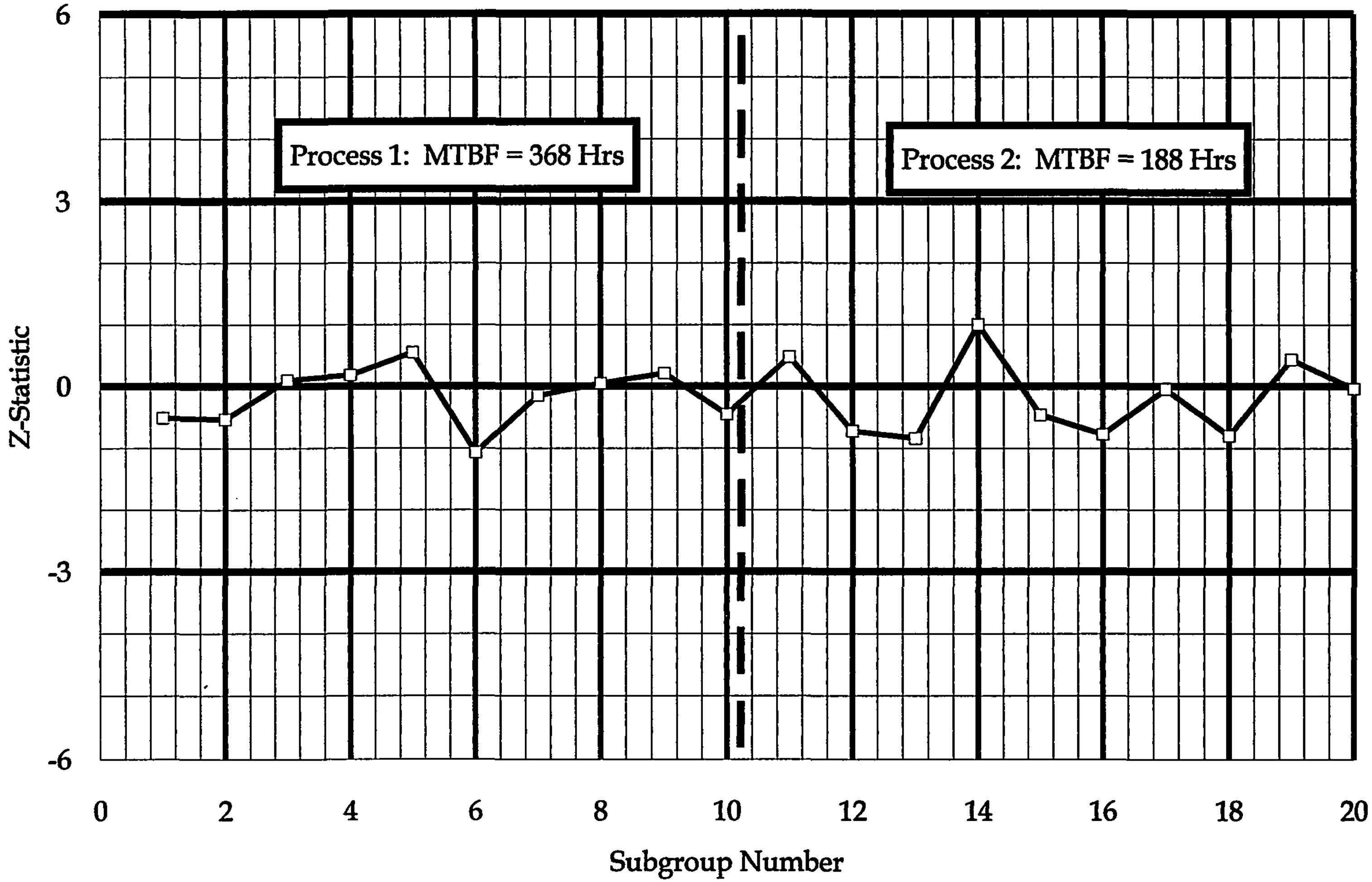

Figure 11. Normalized LRTC for 400 Hour Simulation with Shift (Centered)

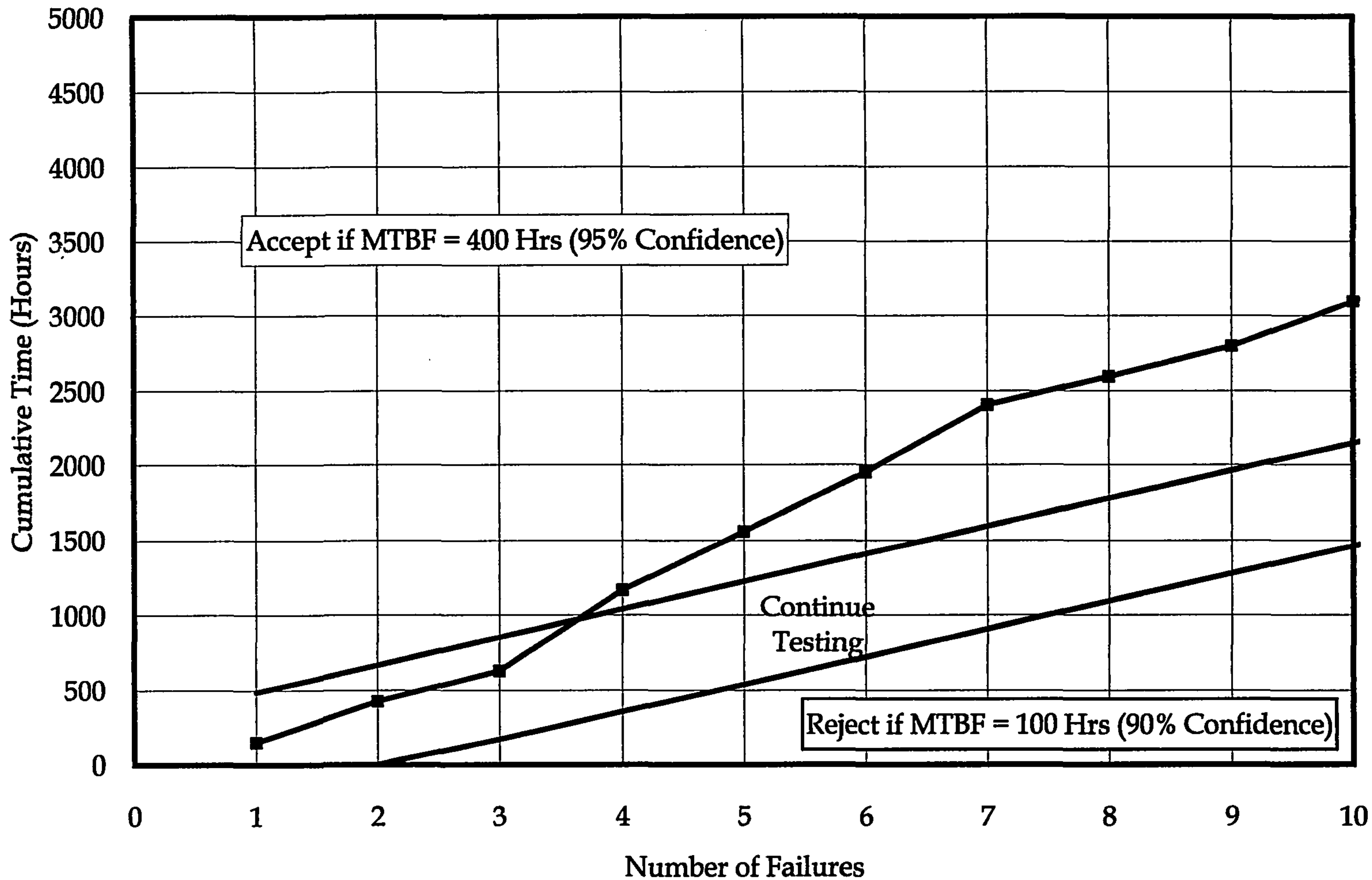

Figure 12. Sequential Life Test for 400 Hour Simulation with Shift

TABLE 6

χ^2 TEST FOR $H0_6$

	TYPE II error	Null	Total	Q value
SEQ	115	0	115	125.0
LRTC	34	81	115	
Total	149	81	230	

The value of the test is 125.0 which is above the value for the χ^2 with 3 degrees of freedom at 97.5% statistical confidence, therefore $H0_6$ is rejected. The LRTC is superior to the Sequential Life test in avoiding a Type II error ($H0_6$), in avoiding a Type I error ($H0_2$), and in estimating MTBF ($H1_1$).

Duane Model for Reliability Growth Testing

For the process shift case, the Duane model shows two distinct processes (figure 13). In the first process, reliability begins at 191 hours MTBF and increases at a constant rate for 42 failures, until it reaches approximately 400 hours MTBF. From failure 43 until failure 100, reliability begins at 400 hours MTBF and degrades to 270 hours MTBF. The slopes for the two periods are 0.154 and -0.75, respectively. The R^2 values are 0.97 and 0.81, respectively.

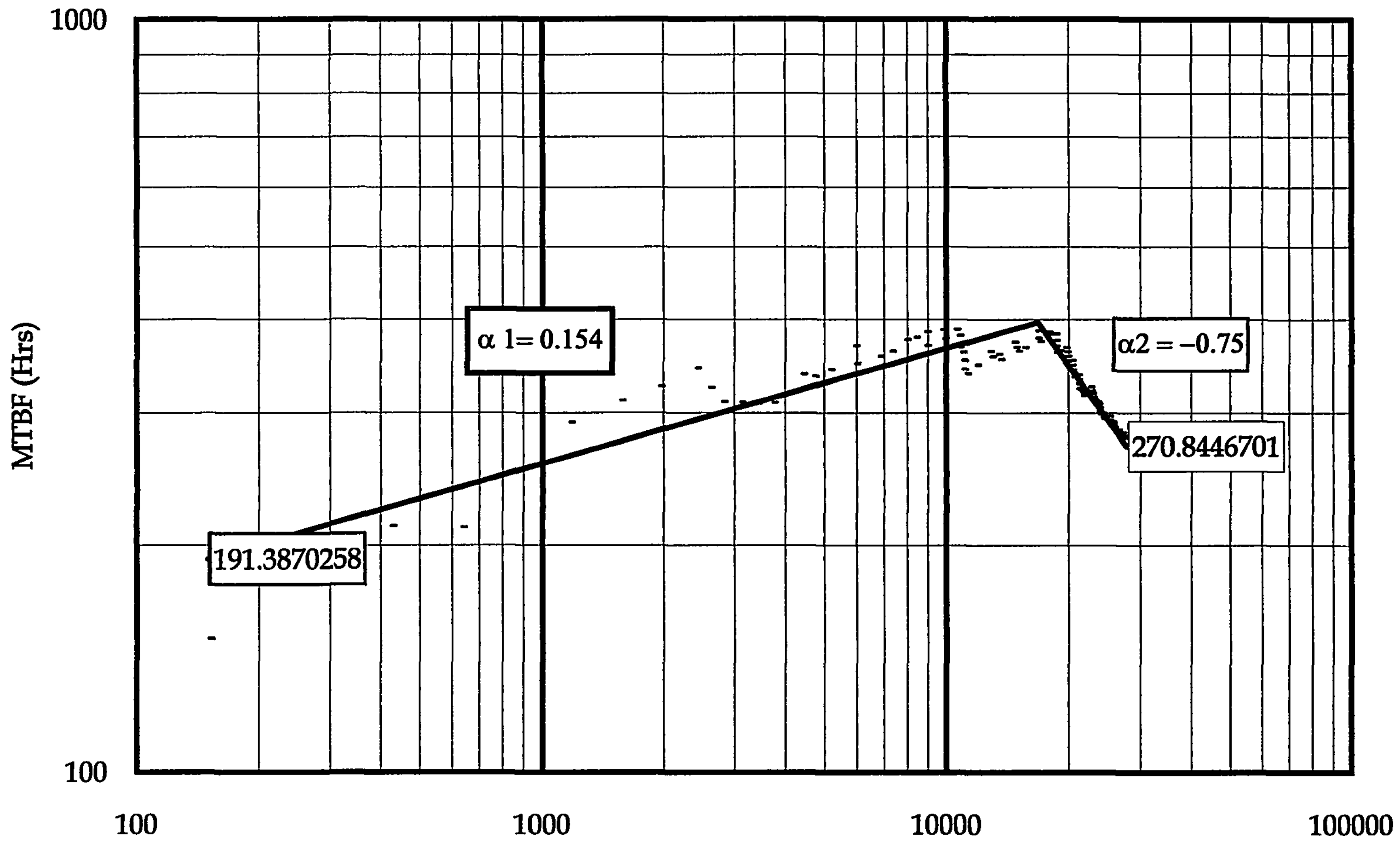

Figure 13. Duane Model for 400 Hour Simulation with Shift

The implication of such high R^2 values is that, with a great deal of certainty, the observed reliability growth and subsequent degradation is founded in fact. In reality, the system neither gradually grew nor did it gradually decrease in reliability. Instead, the system instantaneously decreased in reliability at a discrete point in time and remained at the lower value throughout time.

Comparison of Duane Reliability Growth Model to LRTC, Hypothesis $H0_7$

The Duane Reliability Growth model was performed on 115 sets of random samples from a failure distribution with an MTBF of 400 hours which shifted to 200 hours in period 50. In these 115 tests, 22 were able to detect the change in process. Similarly, the same 115 tests were ran for the LRTC model. Of the 115 tests, 34 were rejected because they appeared to exhibit one of the patterns for an out of control condition (table 7).

TABLE 7

χ^2 TEST FOR $H0_7$

	TYPE II error	Null	Total	Q value
Duane model	93	22	115	61.2
LRTC	34	81	115	
Total	127	103	130	

The value of the test is 61.2 which is above the value for the χ^2 with 3 degrees of freedom at 97.5% statistical confidence, therefore $H0_7$ is rejected. The LRTC is superior to the Duane Reliability Growth model in avoiding a Type II error ($H0_7$), in avoiding a Type I error ($H0_3$), and in estimating MTBF ($H1_1$).

Posterior Test Using Cumulative Mean Time Between Failures

Referring to figure 14, the Posterior test shows a process shift at approximately time T = 24000 hours which can be backdated to T = 18000 hours since at T = 24000 hours is outside of 90% probability of the T = 18000 hours estimate. Although it detects a shift in reliability, it is not clear how to handle the shift in terms of reporting an accurate failure rate. It is unreasonable to average the values from the prior distribution into the new MTBF estimate because it is a cumulative test. However, there is not a method to separate the values.

Of course, one could simply disregard all values before the system shift. If one did so there would be an assumption that a common cause of failure and not an outlier caused the systemic anomaly. If an outlier caused the anomaly, then some earlier values could be used to demonstrate system reliability. If all earlier points are disregarded, the advantage of a cumulative estimate is completely lost.

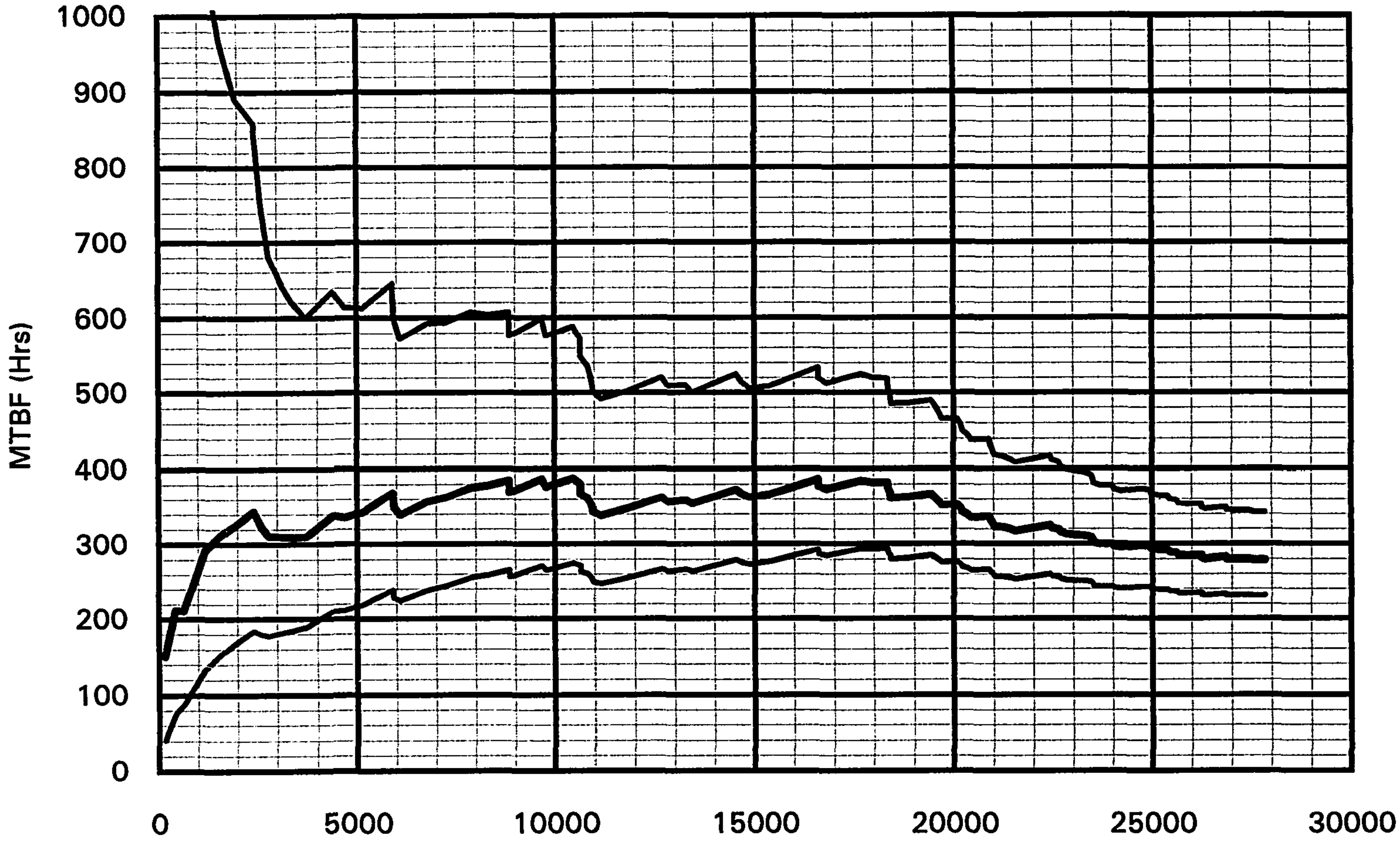

Figure 14. Posterior Test for 400 Hour Simulation with Shift

Comparison of Posterior Test to LRTC, Hypothesis $H0_8$

The Posterior test was performed on 115 sets of random samples from a failure distribution with an MTBF of 400 hours which shifted to 200 hours in period 50. In these 115 tests, none were able to detect the change in process. Similarly, the same 115 tests were ran for the LRTC model. Of the 115 tests, 34 were rejected because they exhibited a Type II error (table 8).

TABLE 8

χ^2 TEST FOR $H0_8$

	TYPE II	Null	Total	Q value
Post. Test	115	0	115	125.0
LRTC	34	81	115	
Total	149	81	130	

The value of the test is 125.0, which is above the value for the χ^2 with 3 degrees of freedom at 97.5% statistical confidence, therefore $H0_8$ is rejected. The LRTC is superior to the Posterior test in avoiding a Type II error ($H0_8$) and in avoiding a Type I error. The Posterior is superior to the LRTC in estimating MTBF of a stable process ($H1_1$).

SPC Charts

Referring to figures 15 and 16, there appears to be a shift in the process at subgroup 9. The value for range, R, at subgroup 9 exceeds 3 standard deviations from the mean of the range. The range chart is "out of control", so the process charts must be redrawn with the anomalous point removed from the calculation. The redrawn or centered charts are shown in figures 17 and 18.

Referring to figures 17 and 18, X-bar appears to be within SPC, meaning that the changes in values from point to point are due solely to random variation in the process and not due to any systemic shift. The R-chart indicates a possible change in variation over time because points five through ten have five points over one standard deviation while all six points are over the mean. Although there appears to be a shift in the range, the x-bar chart does not indicate a companion shift in the averages. The SPC charts would therefore be interpreted in one of the following two ways:

1) The R-chart is out of control, therefore no inferences can be drawn from X-bar chart.

2) The X-bar chart is in control and test by which R chart is found to be out of control is a "weak" test, therefore accept X-bar mean as correct but use caution making any other inferences.

Regardless of the interpretation method used, the precise time of the shift and the companion estimates of the mean are not available.

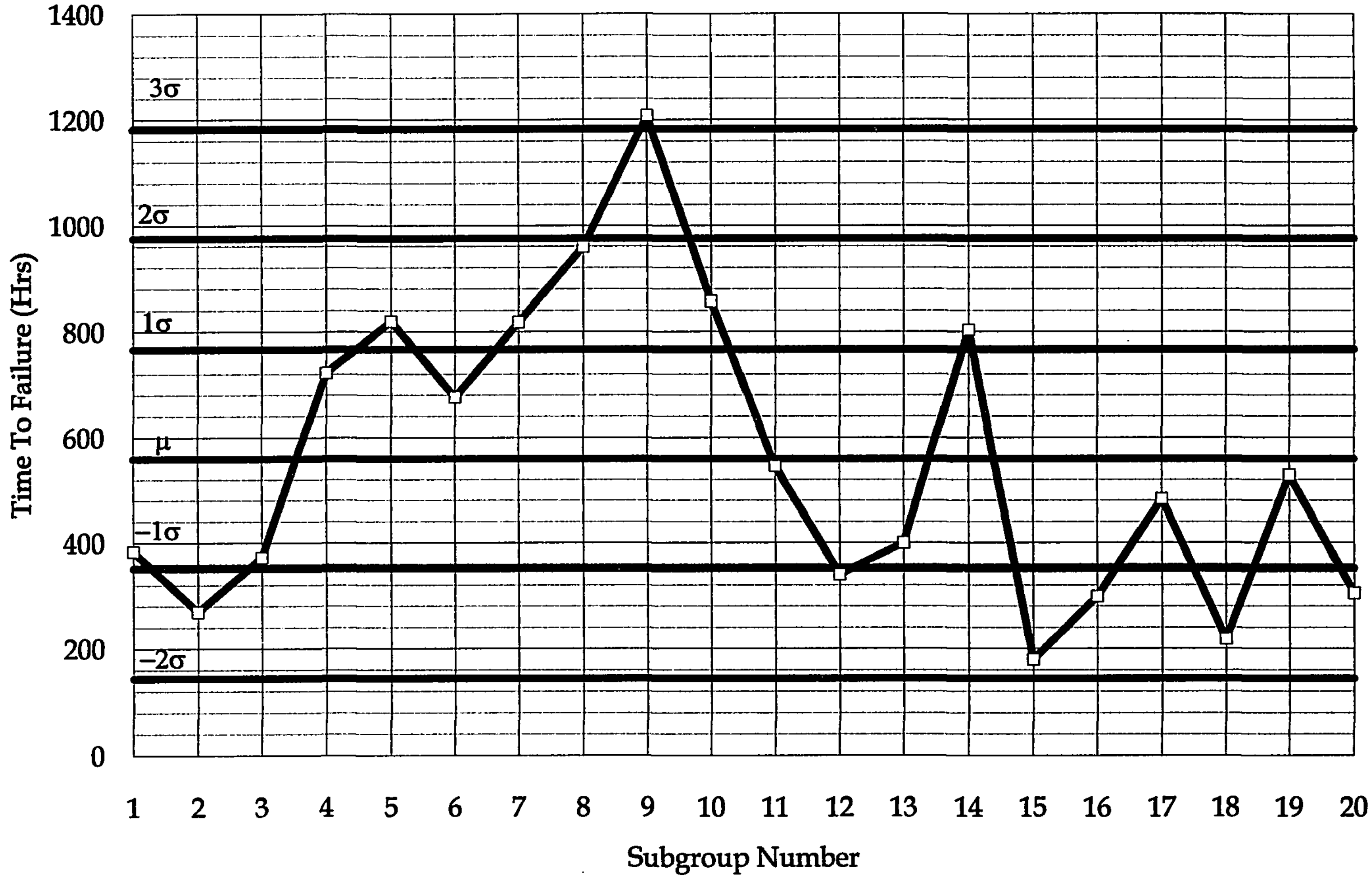

Figure 15. R-Chart for 400 Hour Simulation with Shift

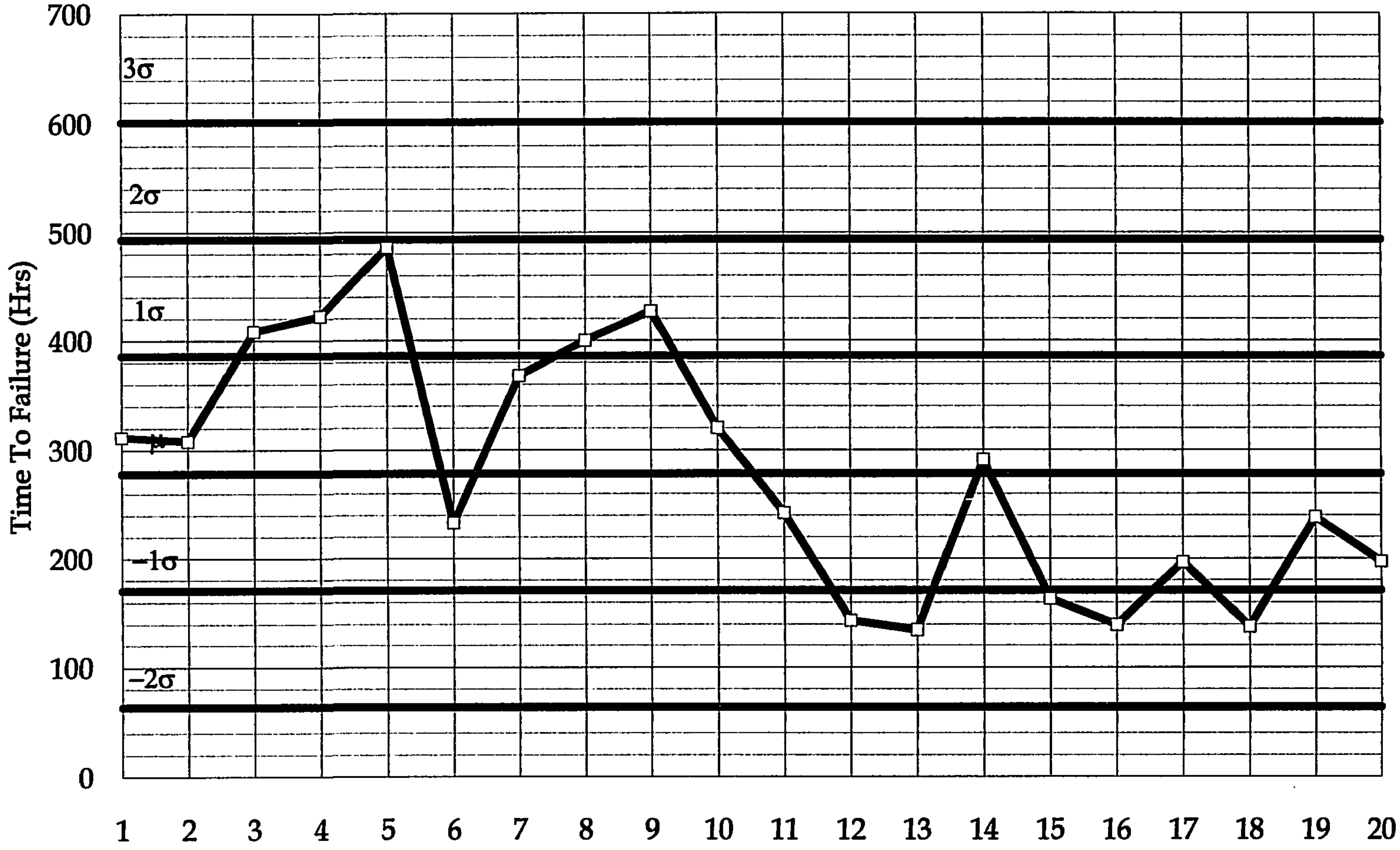

Figure 16. X-Bar Chart for 400 Hour Simulation with Shift

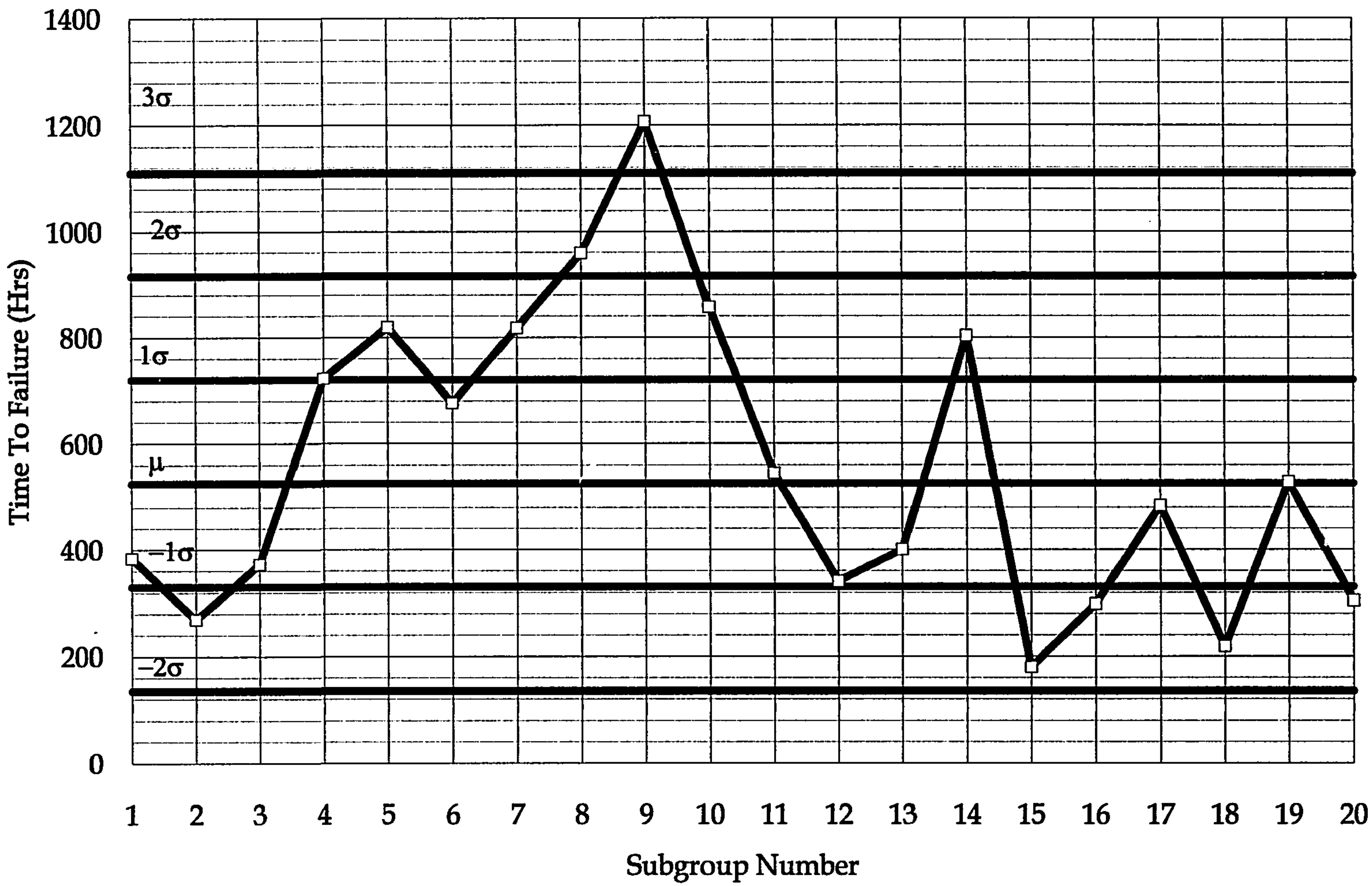

Figure 17. R-Chart for 400 Hour Simulation with Shift (Centered)

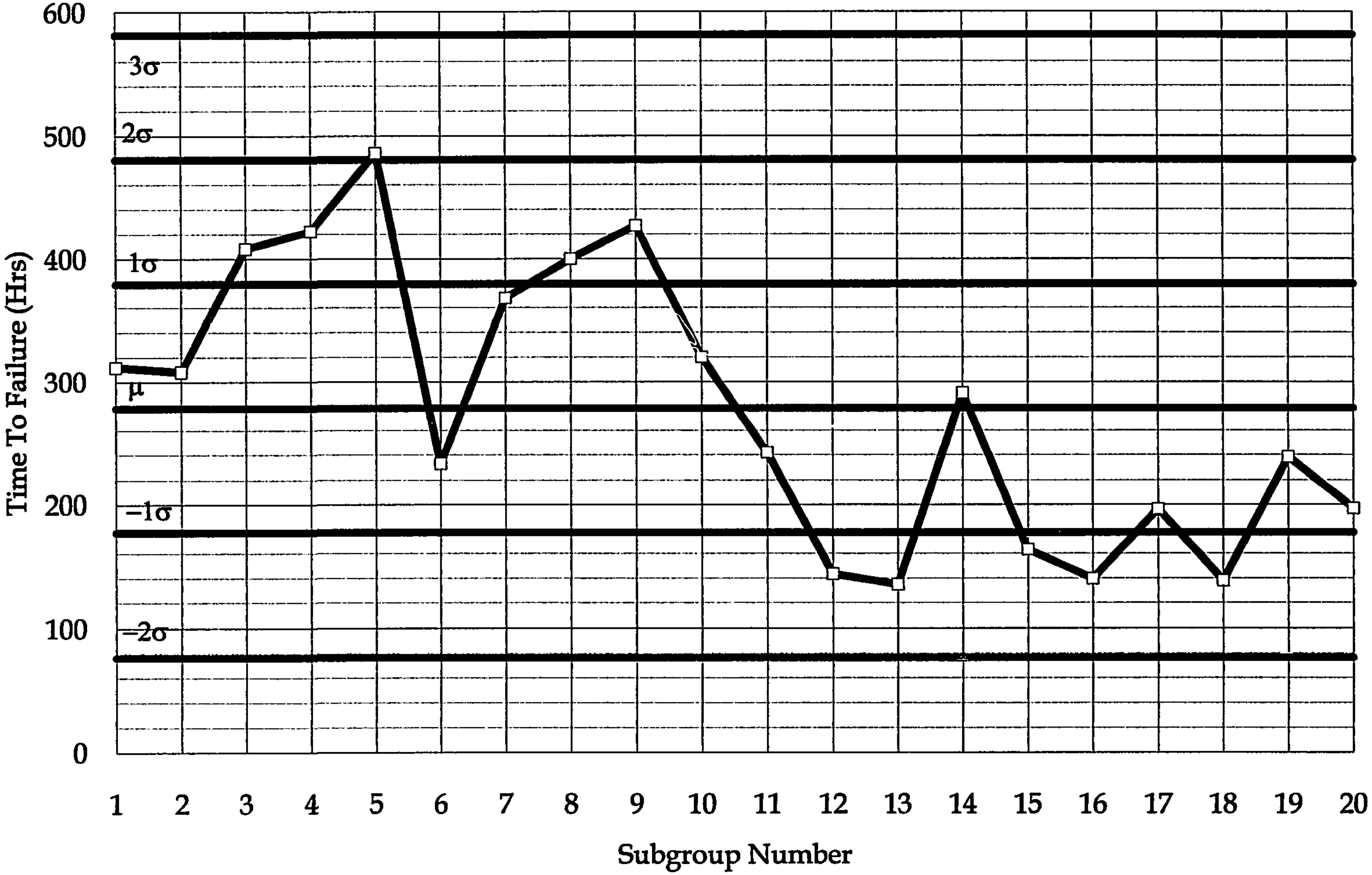

FIG. 18. X-Bar Chart for 400 Hour Simulation with Shift (Centered)

Comparison of SPC to LRTC, Hypothesis $H0_9$

SPC methods were performed on 115 sets of random samples from a failure distribution with an MTBF of 400 hours which shifted to 200 hours in period 50. In these 115 tests, 114 were able to detect the change in process. Similarly, the same 115 tests were ran for the LRTC model. Of the 115 tests, 34 were rejected because they exhibited a Type II error (table 9).

TABLE 9

χ^2 TEST FOR $H0_9$

	TYPE II	Null	Total	Q value
SPC	1	114	115	36.7
LRTC	34	81	115	
Total	35	195	230	

The value of the test is 36.7, which is above the value for the χ^2 with 3 degrees of freedom at 97.5% statistical confidence, therefore $H0_9$ is rejected. SPC is superior to the LRTC to detect if a process has shifted, however, since SPC is more likely to produce a Type I error ($H0_5$), results may be rejected even though an actual change exists. In other words, SPC detects shifts whether or not shifts are present.

Case Studies

Three case studies were selected from fleet failure records. The particular parts which were selected for analysis all have caused significant problems in the fleet leading to a directed study of their reliability. The units are referred to as Power Supply A, Power Supply B, and Power Supply C.

Case Study 1: Power Supply A

Power Supply A is a constant amperage power supply which delivers a constant current, adjustable from 1.8 to 3.5 A over a voltage range of 82.8 to 140 Vdc. A constant current supply is necessary because of possible transmission losses over a very long cable which delivers the signal. The power supply is designed to cut off power if an overvoltage is detected. An overvoltage is any voltage which exceeds 160 ± 5 Vdc.

The power supply was failing at a rate high enough to cause serious logistics concerns for the fleet. An investigation regarding the cause of failure was conducted and the predominant mode of failure was burned out overvoltage circuitry. Although it could not be conclusively proven with failure reports, the only way the failure mode could be reproduced in the laboratory was to trip a circuit breaker after the overvoltage shutdown but before the voltage had been completely bled off the system. A study of maintenance procedures revealed that tests of operability of the system could cause this failure if the operator was unaware that the system was going into overvoltage and panicked when overvoltage shutdown occurred.

Many options for remedy were discussed. The best option was thought to be redesign of the system which was estimated to cost approximately $930,000. A second option was called the "logistics solution", which was to buy a second unit as a spare for each ship and place it onboard. The second option would cost $1,821,000. Because of the high cost of either option, it was decided that the only permissible immediate change was to rewrite the operability test to ensure that the operator did not get too close to the overvoltage limits.

The change to the operability test was implemented in February of 1992. The design problems posed by the original failure mode were never resolved, thus a recommendation to revisit the design option was written in July of 1993. In order to determine whether the design option was still warranted, a new study of failure trends over time was used. The results were plotted on an LRTC.

LRTC on Power Supply A

An LRTC was drawn for Power Supply A for the period of January 1988 through July 1993 (figure 19). Because there was insufficient data to observe trends on a quarterly basis, the data was placed in 1-year moving averages centered on each month. For ease in interpretation, the chart was also normalized (figure 20).

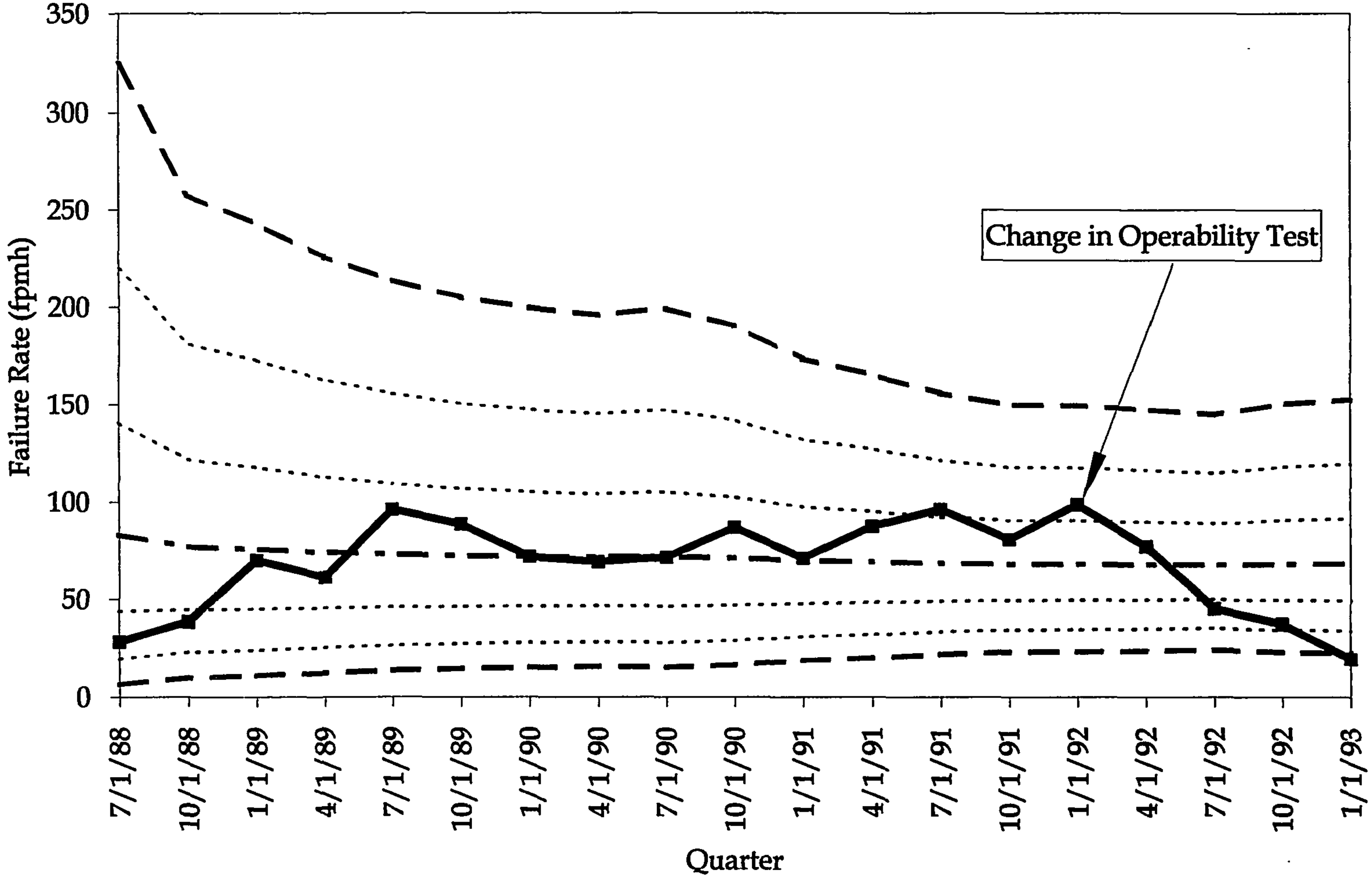

Figure 19. LRTC for Power Supply A

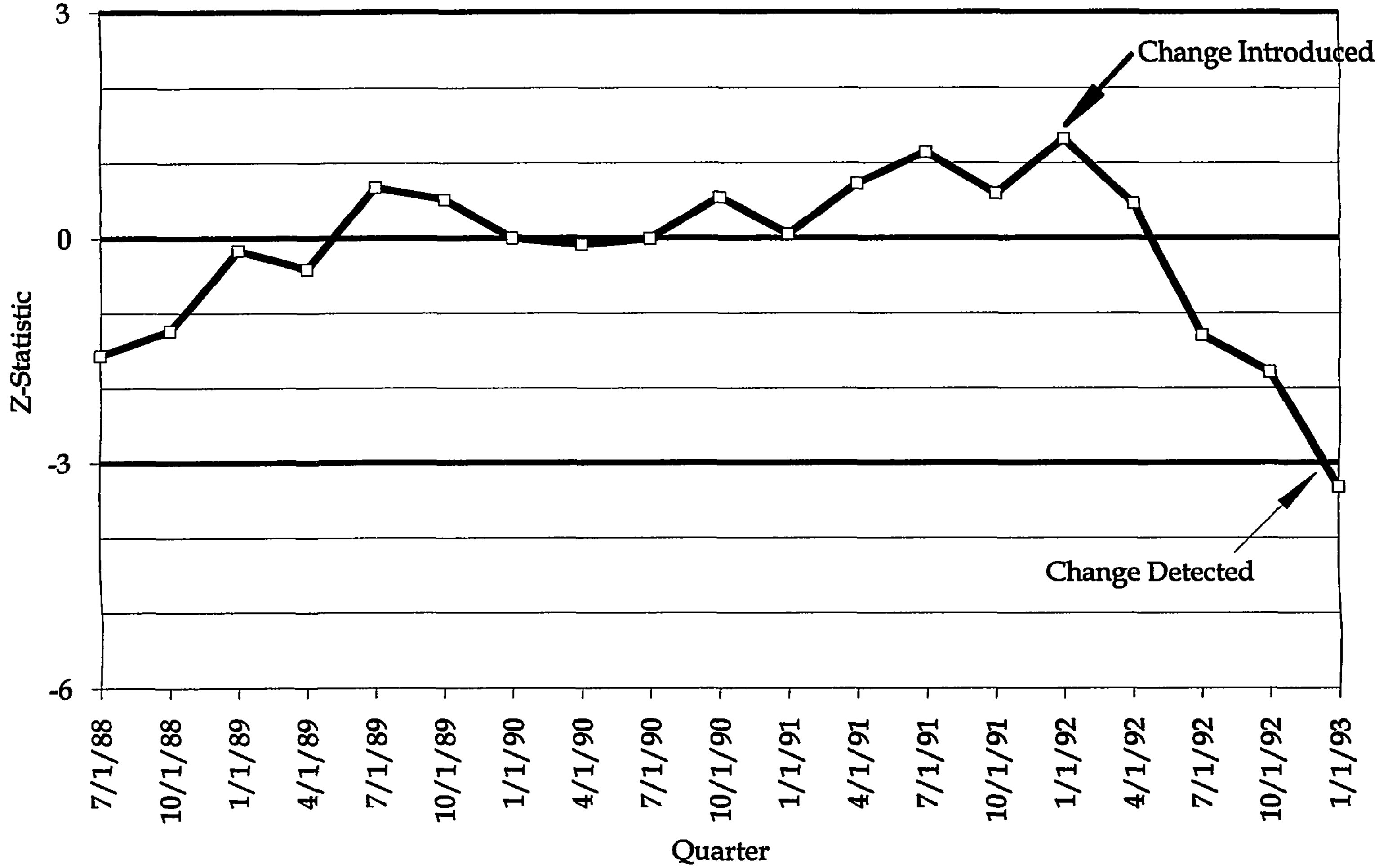

Figure 20. Normalized LRTC for Power Supply A

Referring to figure 20, a meaningful change is detected in the last point of the chart when the failure rate dropped below 99.7% statistical confidence of being within the mean of previous data. Not only was the last point out of control, but also there is a steady decrease in failure rate in each of the data points following the introduction of the change to the system.

Because of the results of the LRTC, it is believed that the reliability of power supply A is improving. Little data is available since the change, so the current failure rate of the power supply cannot yet be established. However, the change appears to be positive and appears to be coincident with the change introduced into the system. It was therefore decided not to redesign the power supply and not to pursue the logistics option at this time. It is estimated therefore that a cost avoidance of between $930,000 (design option) and $2,751,000 (design and logistics options combined) was realized as a direct result of implementation of the LRTC.

Case Study 2: Power Supply B

Power Supply B is a ± 15 Vdc power supply. Recent demands in the fleet had exhausted the supply system of these power supplies and a failure investigation was requested. The LRTC was chosen as the method of analysis for trends of this power supply.

LRTC on Power Supply B

Data was first plotted for Power Supply B on a quarterly basis (figure 21). The LRTC often will demonstrate out of control patterns when no failures occur on a system for an analysis period. This particular system had four time periods in a row with no failures from October 1988 until October 1989. It was therefore decided to use a moving average approach on Power Supply B.

Drawing a 1-year moving average chart (figure 22), centered on each quarter, the LRTC shows a process which has several indications of possible out of control patterns:

1) One point outside of 3z (4/1/89).
2) Two of three points outside of 2z (10/1/88 through 10/1/90).
3) Four out of five points outside of 1z (10/1/88 through 1/1/91).
4) Six or more points in a row increasing (4/1/89 through 10/1/91).
5) Nine points in a row on one side of the centerline (7/1/88 through 7/1/91).

Some of these tests can be confounded when moving averages are used because independence of individual points is lost. However, the weight of so many patterns of out of control conditions, coupled with the fact that they all showed a degenerating process was interpreted as an indication that the power supply was deteriorating in performance.

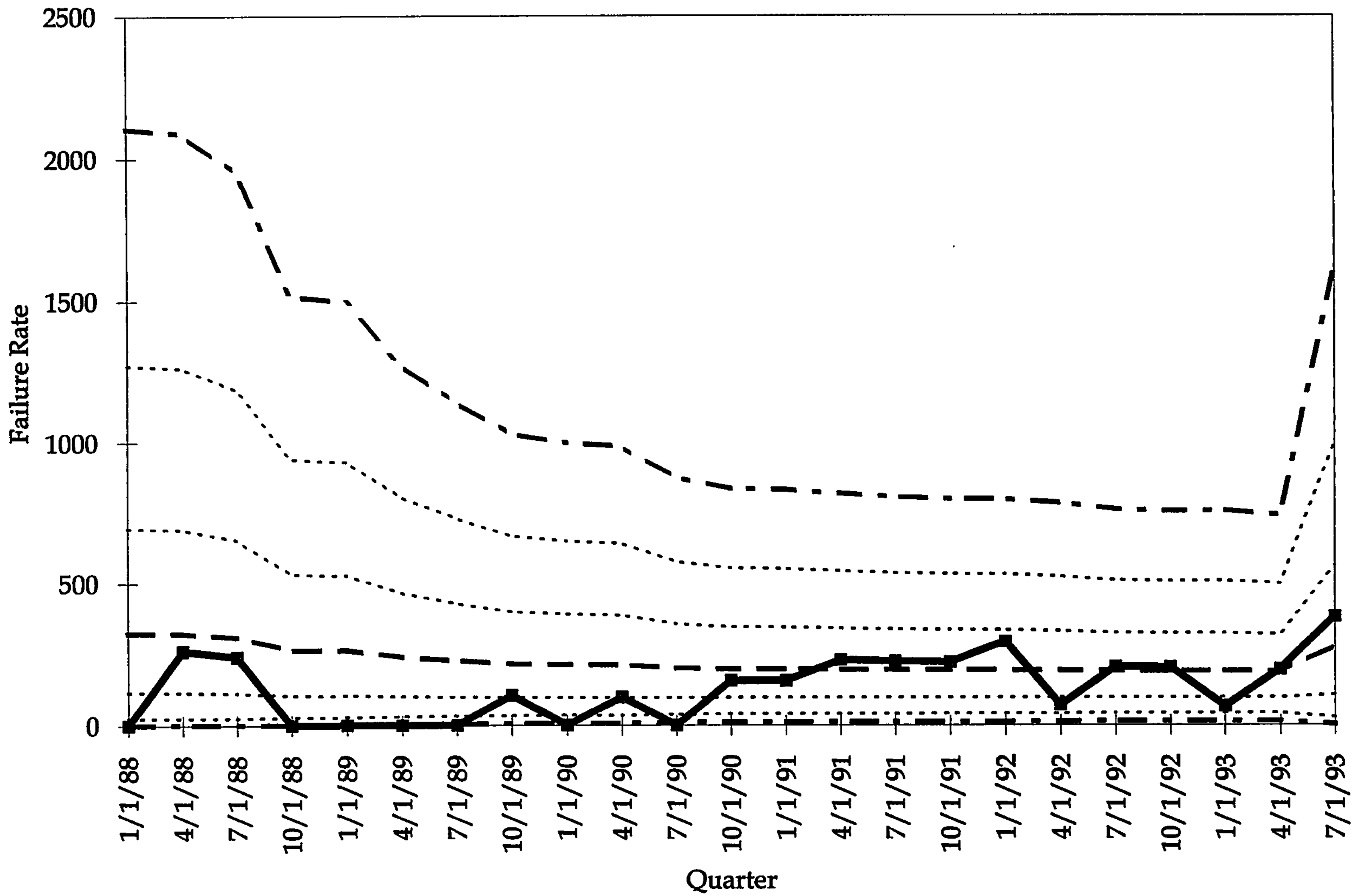

Figure 21. LRTC for Power Supply B

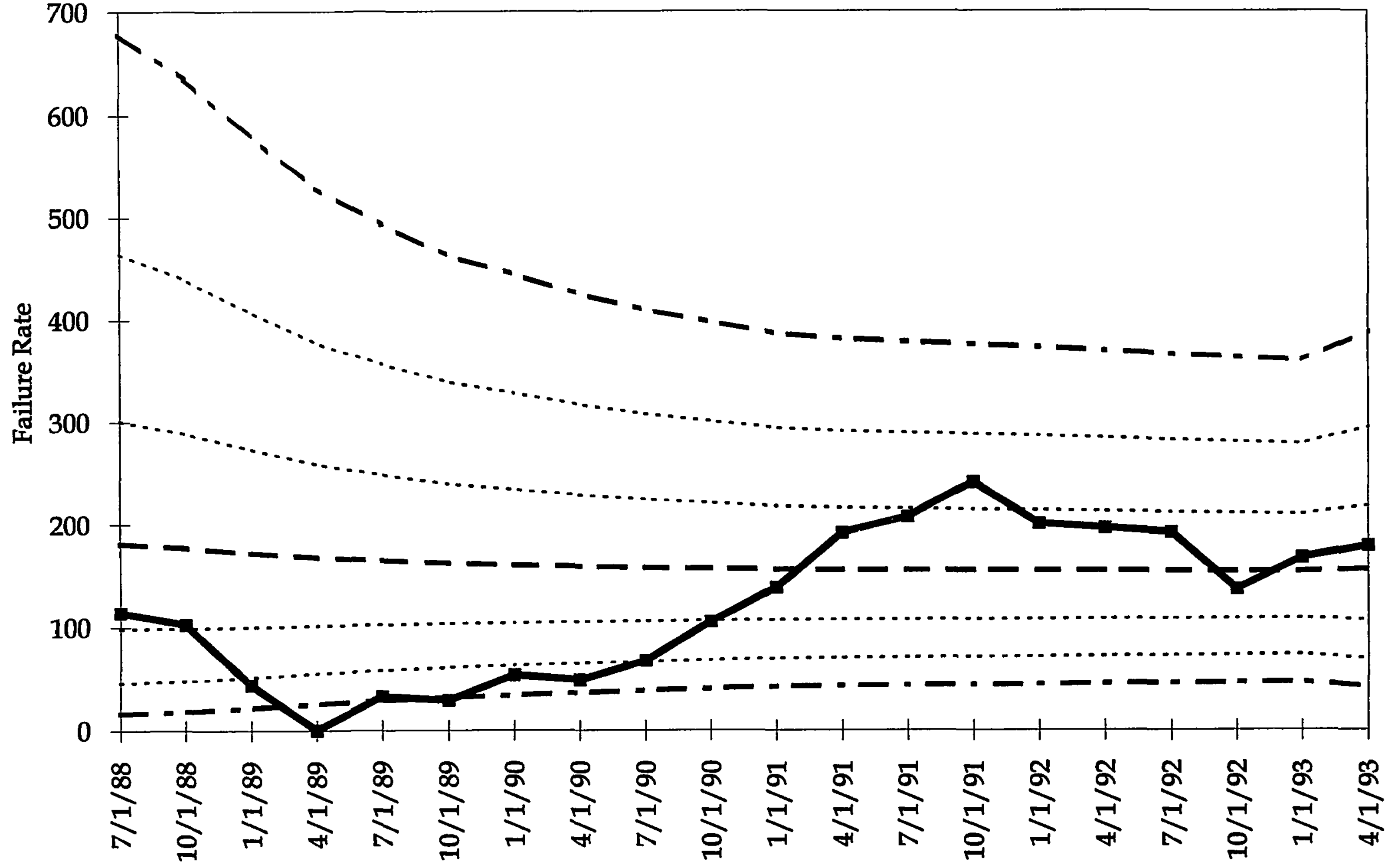

Figure 22. LRTC for Power Supply B (Moving Average)

Case Study 3: Power Supply C

Power Supply C is a constant voltage +5 Vdc Power Supply with 200 A input. A great deal of failures had occurred in the manufacture of these power supplies when an alternate manufacturer tried to produce them. Subsequently, a failure investigation was performed on failed units from the fleet. One symptom was common to many power supplies.

The symptom discovered was a lack of thermal insulation on a circuit card within the power supply for all power supplies manufactured before serial number 200. This lack of thermal insolation was severe because of the high temperatures generated by 200 A of current inside the power supplies. As a result, a recall of the power supplies was ordered. Subject recall cost was to be borne by the contractor.

Due to an administrative problem in the implementation of the recall, the recall was delayed for almost two years. The suppliers were willing to replace the power supplies at no cost, making the government anxious to recall the remaining power supplies from the bad lot. An LRTC was drawn to analyze performance of power supply C in the fleet.

LRTC on Power Supply C

The LRTC (figure 23) yielded interesting results. Referring to the normalized chart (figure 24), one out of control pattern exists in two locations, points three and nine. Moreover, the whole chart appears to show a downward

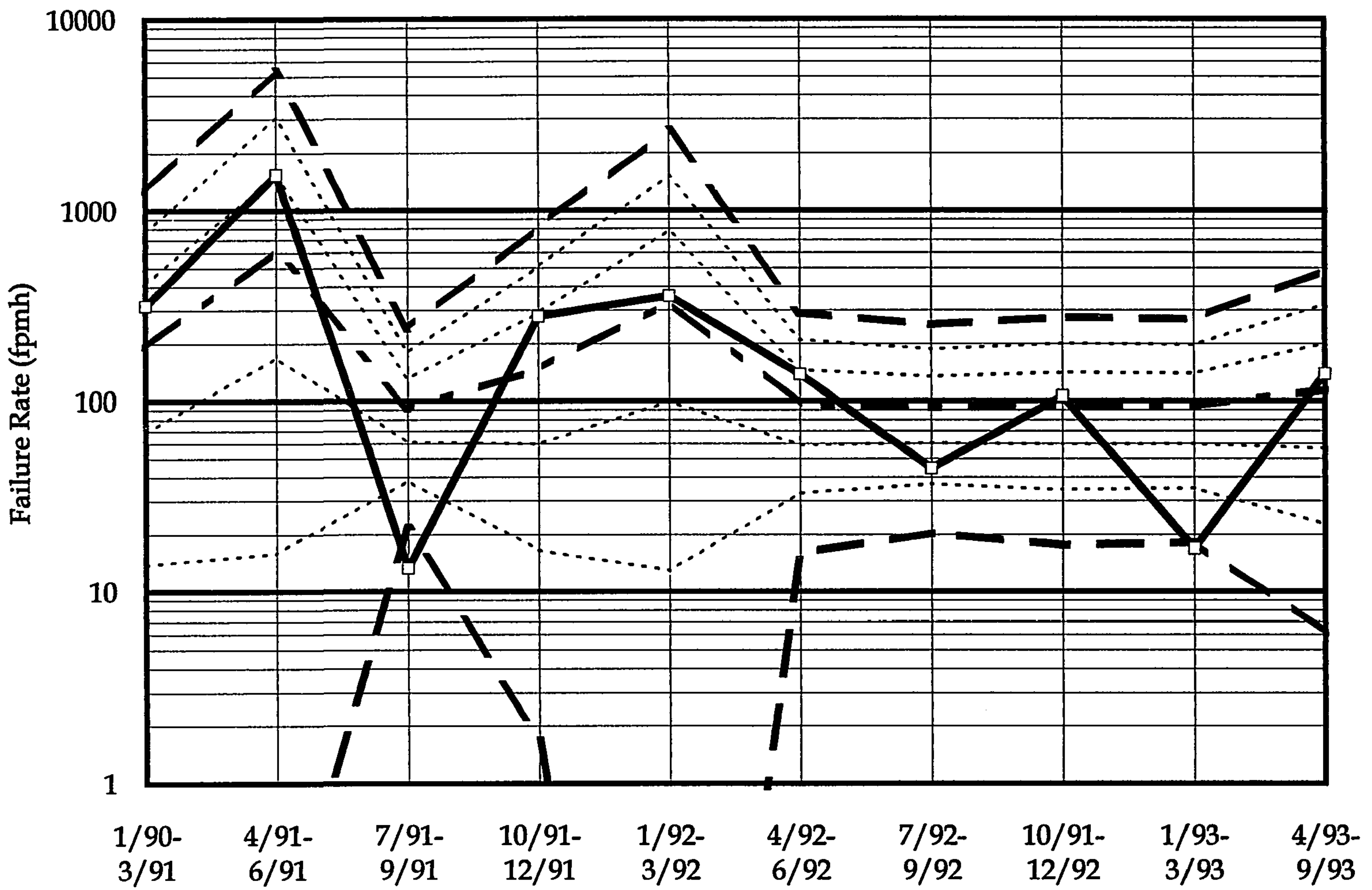

Figure 23. LRTC for Power Supply C

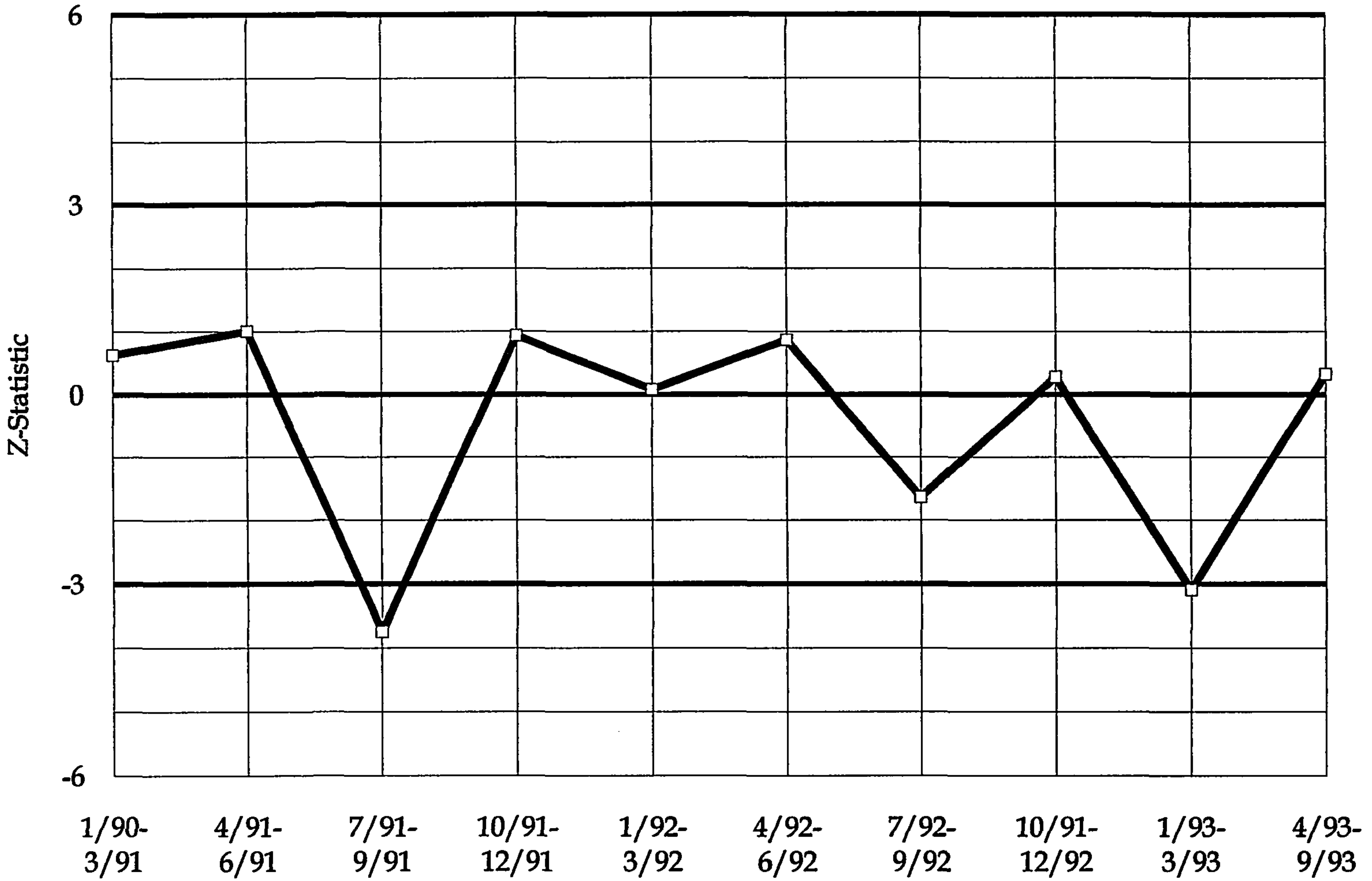

Figure 24. Normalized LRTC for Power Supply C

trend in failure rate which is more evident in figure 23 than in figure 24. The increase in reliability was attributed to infant mortality of the fielded units which lacked thermal compound.

Because an infant mortality trend appears to have occurred, the implementation of the recall program was halted because the units which were bound to fail were already failing and the systems were purging themselves. It was estimated using a Duane model (figure 25) that the system would reach an acceptable level failure rate by mid-1995 with no further interference. The wait option was adopted as the most cost effective method.

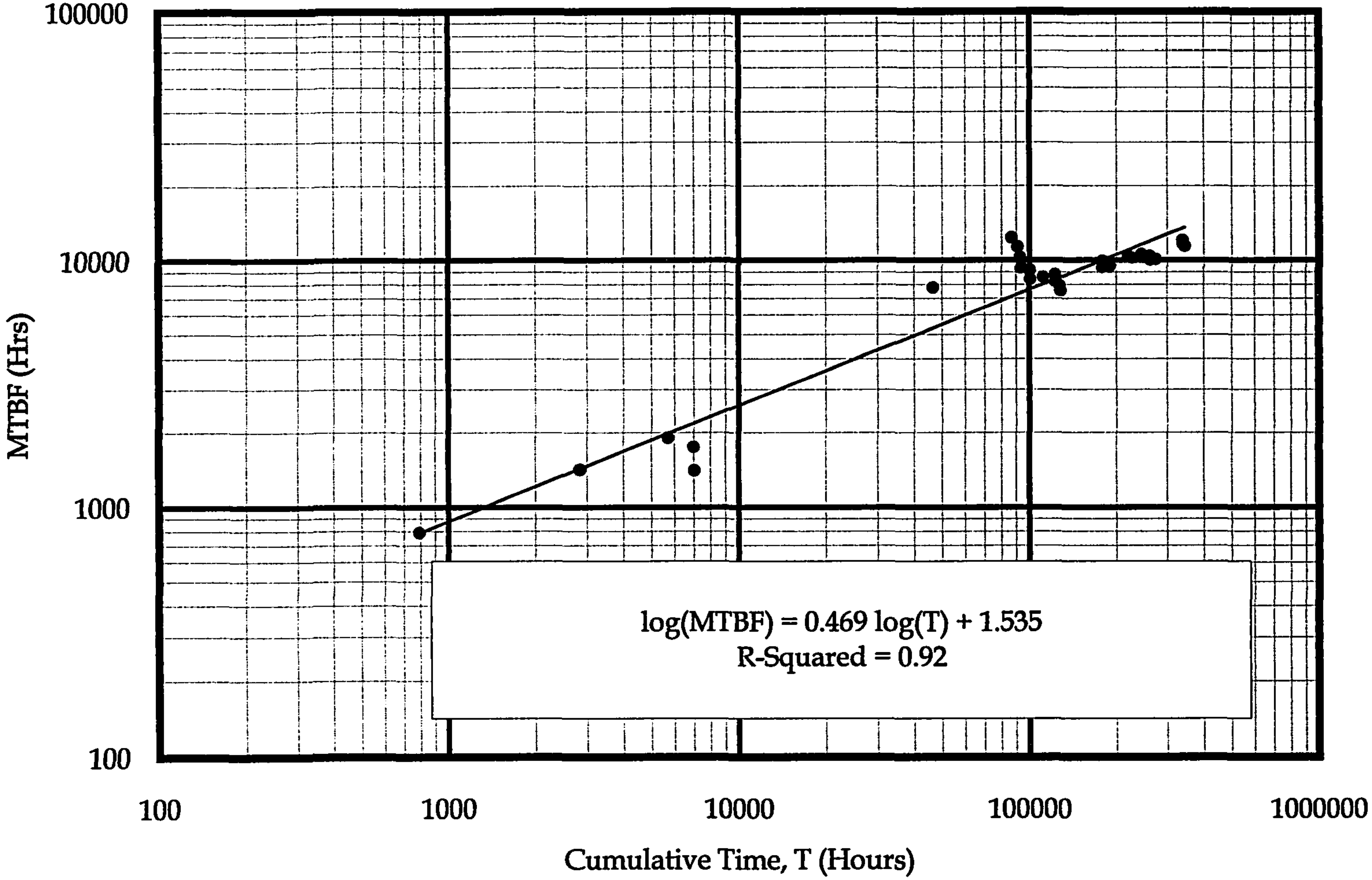

Figure. 25. Duane Reliability Growth Model for Power Supply C

CHAPTER 5

CONCLUSIONS AND FURTHER RESEARCH

Conclusions from Simulations

The objective of the research was to detect changes in the reliability of a target system or part and then estimate an indicator of that reliability. With this in mind, the following conclusions are drawn from the simulations conducted in this research.

The Sequential Life test is not designed to calculate MTBF or to show changes in the system reliability. Instead, it is used to decide whether an existing lot meets specified reliability or not with specified risks. The Sequential Life test is a good method to make accept or reject decisions for homogeneous processes in the shortest amount of time practical. The Sequential Life test is not designed to prevent process from deteriorating. The Sequential Life test detects bad lots and is analogous to the quality sampling procedures, whereas the LRTCs detect real time shifts and is therefore analogous to the preventative Statistical Quality Control (Shewhart) techniques.

Duane models are designed to illustrate whether a production process is producing increasingly higher reliability with time. As such, it assumes that changes in reliability are incremental which is a reasonable assumption during some phases of production. In the simulation, reliability did not gradually grow.

Rather, it grew as a result of a step function applied at a discrete point in time. In field situations where the system is not being subjected to continuous functions like improved production techniques or tighter in-plant quality control, it is not likely that a system will increase in reliability linearly. The Duane model is overly sensitive to outliers in the data, especially if they occur early in the process. Continuous reliability examples were illustrated by the Duane model as either increasing or decreasing in reliability. The information from the Duane model produces the rather disturbing inference that the system will continue to improve along the same patterns in the future.

A step function change is more plausible such as a retrofitted redesign to a part of the system or to a system which interfaces with it. The Duane model was demonstrated to be unable to accurately demonstrate a change due to a step function. The Duane model was able to demonstrate when the step function changes occurred but was unable to accurately predict reliability parameters.

The Posterior test is a method used to develop the best possible reliability parameter estimate from available data. As such, it combines all the data as the data becomes available. If a process does not change with time, the Posterior test becomes more credible as time and failure history accumulates. If a process changes with time, the Posterior test is not adequately equipped to deal with the change. When an outlier exists, a decision must be made to either throw out the outlier or begin accumulating data from the transformed process. Information is not generally known whether one anomalous value is an outlier or the beginning of a new process, therefore the posterior test cannot adequately estimate when shifts occur. The posterior test adequately predicts the reliability parameters of stable systems.

SPC tests are based on the Central Limit Theorem. The Central Limit Theorem states that a distribution of averages will appear normal as the size of the subgroups increase. This poses two problems. First of all, there is generally not much data to build an SPC chart with. Secondly, the Central Limit Theorem works for any process regardless of its original shape and therefore information is lost regarding the shape of a known distribution. Reliabilities of electronic equipment come from exponential distributions, making it reasonable to detect changes in the process using the properties of the exponential distribution.

The results from the SPC studies in the simulations were consistent with the reservations about SPC stated above. Most of the SPC charts drawn from the simulations of exponential distributions showed out of control points when none existed. This excessive "noise" in the SPC chart was due primarily to the shape of the exponential distribution from which the data is derived. The exponential distribution does not have a mode and is strongly skewed to the left. Because of the high probability of lower values for time to failure, the SPC chart would estimate a distribution with a low mean then show higher values as outliers.

In the cases where shifts were introduced, SPC Charts showed shifts in the process. Unfortunately, the SPC charts showed shifts and out of control points so often when no such point existed that even these positive results were inconclusive and hard to interpret correctly. When processes showed control, SPC was adequate to produce mean estimates but even known "in control" processes rarely showed control with SPC techniques.

The LRTC consistently detected shifts in processes at approximately the same point in which the shifts were introduced. Additionally, the LRTCs produced surprisingly accurate results regarding estimates of MTBF and failure rate. Basically, the LRTCs were more successful than the Sequential Life test because they are centered by the actual process values, not predetermined test values and because the LRTCs reassess reliability in each data point, not cumulatively for the whole process. The LRTC was more successful than the Duane model because it was insensitive to random data. The LRTC was more successful than the Posterior test because the LRTC did not depend on homogeneity in the process to produce an accurate prediction. Finally, the LRTC was more successful than SPC techniques because the LRTC was able to successfully exploit the shape of the exponential distribution where SPC relied on the general rule of the Central Limit Theorem.

Conclusions from Case Studies

Results from the case studies validate that the LRTC process is capable of demonstrating when changes occur in the system and give sufficient information to make decisions regarding design options. Power Supply A case study results demonstrate that the LRTC is useful in validation of effects of system changes previously introduced into the system. Power Supply B case study results demonstrate that the LRTC can be used to detect when a system is degrading with time. Power Supply C case study results demonstrate that the LRTC can be interpreted to demonstrate when the affect of infant mortality on a system. Most importantly, case studies A and C demonstrate that conclusions from the LRTC can lead to significant cost avoidance if properly interpreted.

Further Research

The study of the LRTC could be expanded to include several more tests to validate the ability of the LRTC to detect the presence of a ramp function versus a step function change. The LRTC could be tested for its sensitivity against various other discrimination ratios besides the ratio of 2 used in the study. The LRTC could be tested for smaller data runs since it is theoretically possible to perform LRTC analysis with much smaller data sets. The case studies all used less failures than the 100 points generated by the Random Number Generator. The LRTC could be tested against other models of statistical trend, such as ARIMA, EWMA and SPC u-charts. The LRTC could be adapted to nonexponentially distributed functions, such as the Weibull distribution for which the exponential distribution is a special case. LRTC could be applied in the design stage as a method to control reliability of equipments being designed.

The tests which were done on the LRTC prove conclusively that the LRTC is a viable method to test electronic system reliability as it changes over time.

Results for the LRTC are significant and applicable and can now be applied to the analysis of electronic systems with positive results.

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APPENDIX

SIMULATION FAILURE TIMES

Failure Number, n	Time to Fail, t	Total Time, T
1	150.2	150.2
2	273.36	423.56
3	209.04	632.6
4	534.03	1166.64
5	391.11	1557.75
6	393.67	1951.42
7	455.74	2407.15
8	186.68	2593.84
9	201.04	2794.88
10	301.95	3096.83
11	301.73	3398.56
12	317.97	3716.54
13	674.03	4390.56
14	304.48	4695.04
15	442.43	5137.47
16	760.02	5897.49
17	37.32	5934.81
18	168.3	6103.11

Failure Number, n	Time to Fail, t	Total Time, T
19	673.88	6776.99
20	471.96	7248.96
21	639.89	7888.84
22	434.46	8323.3
23	523.86	8847.19
24	5.83	8852.99
25	825.21	9678.2
26	111.2	9789.39
27	678.4	10467.79
28	185.74	10653.53
29	2.54	10656.07
30	186.67	10842.74
31	80.38	10923.12
32	50.07	10973.19
33	182.54	11155.73
34	658.15	11813.88
35	868.04	12681.92
36	146.4	12828.32
37	454.61	13282.94
38	162.8	13445.74
39	1097.71	14543.45
40	138.59	14682.04
41	191.65	14873.69
42	523.84	15397.53
43	1209.33	16606.87
44	2.33	16609.2

Failure Number, n	Time to Fail, t	Total Time, T
45	207.63	16816.83
46	867.43	17684.26
47	284.25	17968.51
48	382.85	18351.36
49	10.24	18361.59
50	54.87	18416.46
51	31.4	18447.86
52	879.46	19327.32
53	1121.69	20449.01
54	209.6	20658.61
55	177.65	20836.26
56	155.04	20991.3
57	776.69	21767.99
58	156.13	21924.12
59	96.78	22020.9
60	249.01	22269.91
61	185.36	22455.27
62	877.67	23332.94
63	94.52	23427.46
64	112.96	23540.42
65	78.24	23618.66
66	459.39	24078.05
67	348.65	24426.7
68	258.77	24685.47
69	1723.89	26409.36
70	117.4	26526.76

Failure Number, n	Time to Fail, t	Total Time, T
71	341.44	26868.2
72	114.76	26982.96
73	277.5	27260.46
74	424.52	27684.98
75	473.59	28158.57
76	332.03	28490.6
77	92.21	28582.82
78	38.81	28621.63
79	296.34	28917.97
80	634.88	29552.85
81	108.21	29661.06
82	325.88	29986.94
83	980	30966.94
84	537.95	31504.89
85	13.1	31517.99
86	342.83	31860.82
87	511.15	32371.97
88	86.62	32458.59
89	364.01	32822.6
90	74.53	32897.13
91	431.95	33329.08
92	713.35	34042.43
93	1.31	34043.74
94	177.78	34221.51
95	1057.88	35279.39
96	29.41	35308.8

Failure Number, n	Time to Fail, t	Total Time, T
97	452.77	35761.57
98	639.33	36400.9
99	264.86	36665.76
100	584.15	37249.91

AUTOBIOGRAPHICAL STATEMENT

Stephen R. Luke is a registered professional engineer in the Commonwealth of Virginia. Mr. Luke received his B.S. in Mechanical Engineering from Virginia Polytechnic Institute in 1983. Mr. Luke is experienced as a test engineer at the Norfolk Naval Shipyard from (1978-1989) and as a quality/reliability engineer at Naval Undersea Warfare Center Detachment (1989-Present). Currently, he is the group leader of the RMA/QA group for the AN/SQQ-89(V) Anti-Submarine Warfare System. Mr. Luke is certified by the American Society for Quality Control as a quality engineer and as a reliability engineer.